Modeling stock-oil co-dependence with Dynamic Stochastic MIDAS Copula models: Online Appendix

Hoang Nguyen *†

Audronė Virbickaitė ‡

Abstract

Stock and oil relationship is usually time-varying and depends on the current economic conditions. In this study, we propose a new Dynamic Stochastic Mixed data sampling (DSM) copula model, that decomposes the stock-oil relationship into a short-run dynamic stochastic component and a long-run component, governed by related macro-finance variables. Inference and prediction is carried out using a novel Bayesian estimation strategy, that can efficiently estimate the latent states and delivers an estimate of the log marginal likelihood used for model comparison. We find that inflation/interest rate, uncertainty and liquidity factors are the main drivers of the long-run co-dependence. We show that the multi-step-ahead variance covariance forecasts constructed using the proposed approach are closer to the true values as compared to the benchmark model. Finally, investment portfolios, based on the proposed DSM copula model, are more accurate and produce better economic outcomes as compared to other alternatives.

Keywords: Copula, Hedging, MIDAS, Portfolio, SMC, Stock-Oil.

JEL Classification: C32, C52, C58, G11, G12

*Corresponding author

†School of Business - Örebro University, 70281, Sweden, hoang.nguyen@oru.se

‡Department of Quantitative Methods, CUNEF Universidad, 28040 Madrid, Spain, audrone.virbickaite@cunef.edu

A Copulas

Here we summarize the CDFs, PDFs and the relationship between the copula-specific parameter θ and the Kendall's tau τ_κ for the bivariate Gaussian, Student-t, Clayton, Gumbel, Frank and Joe copulas (Joe, 2015).

Bivariate Gaussian copula:

Let $x = \Phi^{-1}(u)$ and $y = \Phi^{-1}(v)$, where Φ^{-1} is the inverse of the univariate standard normal distribution function. The bivariate Gaussian copula CDF and PDF are given by:

$$\begin{aligned} C(u,v;\theta) &= \Phi_2\left(x,y;\theta\right), \quad 0<u,v<1, \quad \theta\in[-1,1], \\ c(u,v;\theta) &= \frac{\phi_2\left(\Phi^{-1}(u),\Phi^{-1}(v);\theta\right)}{\phi\left(\Phi^{-1}(u)\right)\cdot\phi\left(\Phi^{-1}(v)\right)} = (1-\theta^2)^{-1/2}\exp\left\{\frac{2\theta xy-\theta^2(x^2+y^2)}{2(1-\theta^2)}\right\}, \end{aligned}$$

where $\Phi_2(\cdot;\theta)$ is the standard bivariate normal distribution function with correlation coefficient θ. The relationship between Kendall's tau and copula parameter is given by:

$$\tau_\kappa = 2\arcsin(\theta)/\pi, \qquad \theta = \sin(\pi\tau_\kappa/2).$$

Bivariate t copula:

Let $x = T_\nu^{-1}(u)$ and $y = T_\nu^{-1}(v)$, where T_ν^{-1} is the inverse of the univariate t-distribution function with ν degrees of freedom. The bivariate t copula CDF and PDF are given by:

$$\begin{aligned} C(u,v;\theta,\nu) &= T_{2,\nu}(x,y;\theta), \quad 0<u,v<1, \quad \theta\in[-1,1], \\ c(u,v;\theta,\nu) &= \frac{t_{2,\nu}(x,y;\theta)}{t_\nu(x)\cdot t_\nu(y)} = \frac{1}{\sqrt{1-\theta^2}}\frac{\Gamma((\nu+2)/2)\Gamma(\nu/2)}{\Gamma^2((\nu+1)/2)}\frac{\left(1+\frac{x^2+y^2-2\theta xy}{\nu(1-\theta^2)}\right)^{-(\nu+2)/2}}{\left(1+\frac{x^2}{\nu}\right)^{-(\nu+1)/2}\left(1+\frac{y^2}{\nu}\right)^{-(\nu+1)/2}}, \end{aligned}$$

where $T_{2,\nu}(\cdot;\theta)$ is the bivariate t distribution function with correlation coefficient θ and degrees of freedom parameter ν. The relationship between Kendall's tau and copula parameter is the same as in Gaussian copula and is given by:

$$\tau_\kappa = 2\arcsin(\theta)/\pi, \qquad \theta = \sin(\pi\tau_\kappa/2).$$

Bivariate Clayton copula:

The CDF and PDF for Clayton copula are given by:

$$C(u,v;\theta) = \left(u^{-\theta}+v^{-\theta}-1\right)^{-1/\theta}, \quad 0\le u,v\le 1, \quad 0\le\theta<\infty,$$

$$c(u,v;\theta) = (1+\theta)[uv]^{-\theta-1}\left(u^{-\theta}+v^{-\theta}-1\right)^{-2-1/\theta}.$$

The relationship between Kendall's tau and copula parameter is given by:

$$\tau_\kappa = \frac{\theta}{\theta+2}, \qquad \theta = \frac{2\tau_\kappa}{1-\tau_\kappa}.$$

Bivariate Gumbel copula:

Let $x = -\log u$ and $y = -\log v$. The CDF and PDF are given by:

$$C(u,v;\theta) = \exp\left\{-\left(x^\theta + y^\theta\right)^{1/\theta}\right\}, \quad 0 \le u,v \le 1, \quad 1 \le \theta < \infty,$$
$$c(u,v;\theta) = (uv)^{-1}\cdot\exp\left\{-[x^\theta+y^\theta]^{1/\theta}\right\}\left[(x^\theta+y^\theta)^{1/\theta}+\theta-1\right]\left[x^\theta+y^\theta\right]^{1/\theta-2}(xy)^{\theta-1}.$$

The relationship between Kendall's tau and copula parameter is given by:

$$\tau_\kappa = \frac{\theta-1}{\theta}, \qquad \theta = \frac{1}{1-\tau_\kappa}.$$

Bivariate Frank copula:

The CDF and PDF are given by:

$$C(u,v;\theta) = -\theta^{-1}\log\left(\frac{1-e^{-\theta}-(1-e^{-\theta u})(1-e^{-\theta v})}{1-e^{-\theta}}\right), \quad 0 \le u,v \le 1, \quad -\infty < \theta < \infty,$$
$$c(u,v;\theta) = \frac{\theta(1-e^{-\theta})e^{-\theta(u+v)}}{[1-e^{-\theta}-(1-e^{-\theta u})(1-e^{-\theta v})]^2}.$$

The relationship between Kendall's tau and copula parameter is given by:

$$\tau_\kappa = 1 + 4\theta^{-1}\left[\theta^{-1}\int_0^\theta \frac{t}{(e^t-1)}dt - 1\right].$$

Bivariate Joe copula:

Let $x = 1-u$ and $y = 1-v$. The CDF is given by:

$$C(u,v;\theta) = 1-\left(x^\theta+y^\theta-x^\theta y^\theta\right)^{1/\theta}, \quad 0 \le u,v \le 1, \quad 1 \le \theta < \infty,$$
$$c(u,v;\theta) = (x^\theta+y^\theta-x^\theta y^\theta)^{-2+1/\theta}x^{\theta-1}y^{\theta-1}[\theta-1+x^\theta+y^\theta-x^\theta y^\theta].$$

The relationship between Kendall's tau and copula parameter is given by:

$$\tau_\kappa = 1 + 2(2-\theta)^{-1}[\text{digamma}(2) - \text{digamma}(2/\theta + 1)]$$

There is no closed form expression between θ and τ_κ (numerical inversion).

Rotation of Bivariate Archimedean copulas

The CDF, PDF of the 90-degree, 180-degree, 270 degree rotation of Clayton, Gumbel, and Joe copula model can be derived from its CDF and PDF,

$$\begin{aligned}
C_{90}(u,v;\theta) &= v - C(1-u,v;-\theta),\\
C_{180}(u,v;\theta) &= u + v - 1 + C(1-u,1-v;\theta),\\
C_{270}(u,v;\theta) &= u - C(u,1-v;-\theta),\\
c_{90}(u,v;\theta) &= c(1-u,v;-\theta),\\
c_{180}(u,v;\theta) &= c(1-u,1-v;\theta),\\
c_{270}(u,v;\theta) &= c(u,1-v;-\theta).
\end{aligned}$$

B DTSMC algorithm

1. Sample $\Theta_0^j \sim p(\Theta)$, $e_0^j \sim p(e)$ and set $W_0^j = 1/M$ for $j = 1...M$ with $M = 1000$ particles
2. For $i = 1, ..., S$, with $S = 10000$ level temperatures,

   Step 1: Reweighting: Calculate the unnormalized weights and the normalized weights

$$w_i^j = W_{i-1}^j \frac{\widehat{p}(u_{1:T}|\Theta_{i-1}^j, e_{i-1}^j)^{\gamma_i} p(\Theta_{i-1}^j)}{\widehat{p}(u_{1:T}|\Theta_{i-1}^j, e_{i-1}^j)^{\gamma_{i-1}} p(\Theta_{i-1}^j)} = W_{i-1}^j \widehat{p}(u_{1:T}|\Theta_{i-1}^j, e_{i-1}^j)^{\gamma_i - \gamma_{i-1}}, \quad j = 1, ..., M,$$

$$W_i^j = \frac{w_i^j}{\sum_{s=1}^M w_i^s}, \quad j = 1, ..., M.$$

   Step 2: Calculate the effective sample size (ESS): $\text{ESS} = \frac{1}{\sum_{j=1}^M (W_i^j)^2}$.

   if $\text{ESS} < cM$ for $c = 0.8$

   (i) Resampling: Resampling from $\{\Theta_{i-1}^j, e_{i-1}^j\}_{j=1}^M$ using the weights $\{W_i^j\}_{j=1}^M$, and then set $W_i^j = 1/M$ for $j = 1...M$, to obtain the new equally-weighted particles $\{\Theta_i^j, e_i^j, W_i^j\}_{j=1}^M$.

   (ii) Markov move: Parallel for each $j = 1, ..., M$, move the samples Θ_i^j, e_i^j for $Q = 10$ CPM steps (Deligiannidis et al., 2018):

   (a) Sample Θ_i^{j*} from the random walk proposal density $q(\Theta_i^{j*}|\Theta_i^j)$.

   (b) Sample $\epsilon^j \sim \mathbf{N}(\mathbf{0}, \mathbf{I})$ and set $e_i^{j*} = \rho e_i^j + \sqrt{1-\rho^2}\epsilon^j$ with $\rho = 0.999$ is a correlation factor.

   (c) Compute the estimated likelihood $\widehat{p}(u_{1:T}|\Theta_i^{j*}, e_i^{j*})$ using a bootstrap particle filter

   (d) Set $\Theta_i^j = \Theta_i^{j*}$ and $e_i^j = e_i^{j*}$ with the probability

$$\min\left(1, \frac{\widehat{p}(u_{1:T}|\Theta_i^{j*}, e_i^{j*})^{\gamma_i} p(\Theta_i^{j*})}{\widehat{p}(u_{1:T}|\Theta_i^j, e_i^j)^{\gamma_i} p(\Theta_i^j)} \frac{q(\Theta_i^j|\Theta_i^{j*})}{q(\Theta_i^{j*}|\theta_i^j)}\right),$$

   otherwise keep Θ_i^j, e_i^j unchanged.

3. The log of marginal likelihood estimate is

$$\log\widehat{p}(u_{1:T}) = \sum_{i=1}^K \log\left(\sum_{j=1}^M w_i^j\right).$$

Algorithm 1: The DTSMC algorithm

C A DCC MIDAS Gaussian copula model

Colacito et al. (2011) and Conrad et al. (2014) extend the DCC model (Engle, 2002) such that there are N variables that can explain the long-term dependence. A DCC MIDAS Gaussian copula model can be presented as,

$$\begin{aligned}(u_{1t}, u_{2t}) &\sim c_{(Gauss)}(u_{1t}, u_{2t}|R_t),\\ R_t &= Q_t^{*-1/2} Q_t Q_t^{*-1/2} \text{ where } Q_t^* = diag(Q_t)\\ q_{12,t+1} &= q_{12,\tau}(1-\alpha-\beta) + \alpha\Phi^{-1}(u_{1t})\Phi^{-1}(u_{2t}) + \beta q_{12,t},\\ q_{12,\tau} &= \lambda_0 + \sum_{j=1}^{N} \delta_j \left[\sum_{k=1}^{K_j} \phi_k(\omega_{j,1}, \omega_{j,2}) X_{j,\tau-k}\right],\end{aligned} \tag{1}$$

where $(\lambda_0, \alpha, \beta, \delta_j, \omega_j)$ are the fixed copula parameters and $\tau = \lfloor t/L \rfloor$. $(X_{1\tau}, \ldots, X_{N\tau})$ are N-dimensional vector of low-frequency variables, and $\phi_k(\omega_{j1}, \omega_{j2})$ is the weighting scheme of the variable j on its k lag, for $k = 1, \ldots, K$. The weighting scheme of each variable j depends on the regulated parameter ω_j for $j = 1, \ldots, N$. Note that $0 < \alpha + \beta < 1$.

D Simulation study

In this section, we compare the proposed DSM copula models with the Exponentially Weighted Moving Average (EWMA), the Dynamic Conditional Correlation (DCC) model (Engle, 2002; Tse and Tsui, 2002), and the DSC when the true correlation structure is known, in different stress scenarios based on the proposal of Engle (2002) and (Hafner and Manner, 2012). We simulate $T = 2000$ observations from a bivariate Gaussian copula with time-varying correlation parameter ρ_t. We consider five models for the behavior of ρ_t such that,

1. Constant: $\rho_t = 0.8$.
2. Sine: $\rho_t = 0.5cos(2\pi t/250)$.
3. Fast Sine: $\rho_t = 0.5cos(2\pi t/25)$.
4. Step: $\rho_t = 0.5 - I(t > 1000)$.
5. Ramp: $\rho_t = ((t \mod 200) - 100)/101$.

Engle (2002) considers that these stress tests mimic different realistic contexts that the correlation can be constant, gradual changes, rapid changes, and abrupt changes. We generate 100 datasets for each stress test and obtain the estimate of the correlation process $\hat{\rho}_t$. We calculate the 22-day realized correlation (RCor) as a low-frequency explanatory variable for the long-run change in the correlation. We measure the accuracy of each model based on the mean absolute error (MAE) and the mean-squared error (MSE),

$$MAE = \frac{1}{T}\sum_{t=1}^{T}|\hat{\rho}_t - \rho_t|,$$
$$MSE = \frac{1}{T}\sum_{t=1}^{T}(\hat{\rho}_t - \rho_t)^2.$$

Table 1 compares the relative MAE and MSE of the estimation of the correlation using the EWMA, the DSC, the DSM copula over the benchmark DCC model. With the exception of constant correlation scenarios, the DSM copula has a smallest MAE and MSE due to the ability of capturing the long-run dependence. We also observe that when the correlation changes quickly which makes it hard to extract the long run signal, the DSM copula model is still on par with the DSC model. The choice of lag number of the explanatory variable is also robust to the estimation results.

Table 1: MAE and MSE results: a simulation study

	Constant	Sine	Fast sine	Step	Ramp
(a) MAE					
EWMA	5.316	1.184	1.191	1.000	1.364
DCC	1.000	1.000	1.000	1.000	1.000
DSC	0.796	0.970	0.924	0.980	0.951
DSM	0.902	0.829	0.924	0.875	0.925
(b) MSE					
EWMA	27.679	1.345	1.422	0.864	1.915
DCC	1.000	1.000	1.000	1.000	1.000
DSC	0.618	0.942	0.902	1.006	0.944
DSM	0.798	0.691	0.906	0.855	0.880

The table shows the relative MAE and MSE of the estimation of the correlation using the EWMA, the DSC, the DSM copula over the benchmark DCC model. We use the restricted beta weighting function and $K = 12$ lags of monthly RCor as a low-frequency explanatory variable for the long-run change in the correlation. We generate 100 pseudo datasets for each stress test and calculate the average of MAE and MSE. The entries less than 1 indicate that the given model is better.

E Data description

The daily prices, log returns and monthly correlations for both assets are shown in Figure 1. Shaded areas are the National Bureau of Economic Research (NBER) based recession indicators: early 1990s and 2000s recessions, the global financial crisis of 2008 and the Covid pandemic. As seen from the top right and middle right plots, both asset returns present increased volatility during crisis episodes, which is in line with a well-documented stylized features of financial returns. The bottom plot draws the monthly stock-oil correlation, calculated as a 22-day sample correlation at the end of each period. Just by eye-balling the plot, it seems that the correlation presents a rather mixed picture: sharp decrease during the early 1990s recession, virtually unchanged level during the early 2000s, significant jump-type change in the level during the global financial crisis and rather inconclusive change in the co-dependence during the Covid pandemic. Contrary to the "conventional" asset returns, whose co-dependence increases during economic turmoils and decreases during the periods of calm economic conditions, the stock-oil relationship seems much more complex and possibly driven by a multitude of external factors.

Table 2 presents the descriptive statistics for S&P 500 and WTI returns. Both assets' returns are negatively skewed, have large kurtosis, and present autocorrelation in squared returns, indicating the presence of ARCH effects. The average of daily stock returns is higher than oil returns while oil returns are more volatile and skewed. This could be due to the mismatch of oil supply and demand. Following Hong et al. (2007), the test statistics for the symmetric exceedance correlation between (III-I) quadrants and between (II-IV) quadrants show no statistical evidence of asymmetric tail dependence between S&P 500 and WTI log returns.

Table 2: Descriptive statistics for S&P 500 and WTI log returns.

(a) Summary statistics of S&P 500 and WTI log returns (01/01/1990 - 31/12/2021)

	mean	Q2	sd	skew	kurtosis	min	max	LB(10)	ARCH(10)	JB	n
S&P 500	0.03	0.06	1.14	-0.41	14.39	-12.77	10.96	0.00	0.00	0.00	8059
WTI	0.01	0.05	2.65	-1.87	54.26	-60.17	31.96	0.00	0.00	0.00	8059

(b) Tests for exceedance correlations symmetry of daily log return data in subperiods

Period	$\bar{\rho}$	$\rho^{III}_{0.05}$	$\rho^{I}_{0.05}$	Test (III - I)	P.val	$\rho^{II}_{0.05}$	$\rho^{IV}_{0.05}$	Test (II - IV)	P.val
1990 - 1999	-0.12	0.224	0.468	0.388	0.533	-0.167	-0.581	0.605	0.437
2000 - 2009	0.129	0.224	-0.194	2.974	0.085	-0.139	-0.422	0.886	0.347
2010 - 2021	0.317	0.105	0.057	0.149	0.700	0.442	-0.098	1.788	0.181
1990 - 2021	0.143	0.098	-0.019	1.091	0.296	-0.024	-0.222	1.153	0.283

Panel (a) reports the descriptive statistics for daily log return data (in %) from 01/01/1990 to 31/12/2021. The LB(k) reports the p-value for the Ljung-Box test for autocorrelation of k lags, and ARCH(l) reports the p-value for the test for ARCH effects with l lags. The JB reports the p-value for the Jarque-Bera test for Normality.

Panel (b) reports the Pearson correlation of daily log return data in subperiods. The 5% exceedance correlations at different quadrants are denoted by $\rho^{I}_{0.05}, \rho^{II}_{0.05}, \rho^{III}_{0.05}, \rho^{IV}_{0.05}$. The test statistics for the symmetric exceedence correlation between (III-I) quadrants and between (II-IV) quadrants are calculated following Hong et al. (2007). ***,**,* denote significant at 1%, 5%, 10% level.

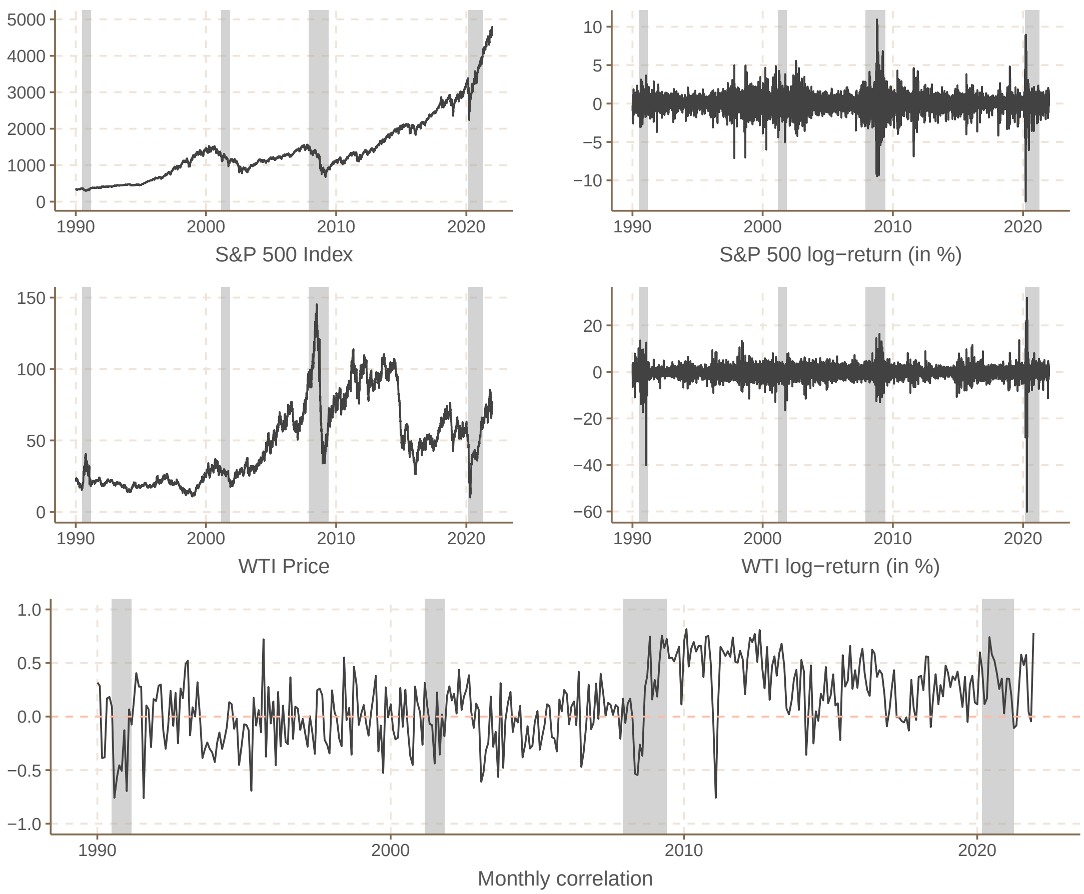

Figure 1: Daily prices, log returns and monthly correlations for S&P 500 and WTI.

The plots show the daily S&P 500 index and returns and the daily WTI oil prices and returns during the period from 01/01/1990 to 31/12/2021. The S&P 500 returns (WTI oil returns) are calculated as the log difference of the S&P 500 index (WTI oil price) multiplied by 100. Monthly correlation is calculated as a 22-day sample correlation at the end of each period. The shaded areas highlight the recession periods based on the NBER recession indicators.

F Marginals

The log returns for asset $i = \{1 \text{ for S\&P } 500, 2 \text{ for WTI}\}$ can be decomposed into time-invariant mean component and a heteroscedastic error term $r_{i,t} - \mu_i \equiv \varepsilon_{i,t} = \epsilon_{i,t}\sqrt{\sigma^2_{i,t}}$. Here $\epsilon_{i,t}$ follows a skew-t distribution with skew and shape parameters (ξ_i, ν_i) (Fernández and Steel, 1998; Ferreira and Steel, 2006). The GJR-GARCH model is defined as follows:

$$\sigma^2_{i,t} = \omega_i + (\alpha_i + \gamma_i I_{i,t-1})\varepsilon^2_{i,t-1} + \beta_i \sigma^2_{i,t-1},$$

where the indicator function $I_{i,t}$ takes value of 1 if $\varepsilon_{i,t} \leq 0$ and 0 otherwise; and $(\omega_i, \alpha_i, \beta_i, \gamma_i)$ are the parameters of the marginal returns where γ_i represents the leverage effect. The models were estimated using the `R` package `rugarch` (Ghalanos, 2022). Estimation results are in Table 3. For both assets the persistence parameter β_i takes a value close to one, a finding consistent with the stylized features of financial volatility. The leverage parameter γ_i is larger for S&P 500 than for WTI indicating a stronger asymmetric response of the volatility, and both parameters are statistically significant. The degrees of freedom parameter ν_i is relatively low in both cases, indicating fat-tailed distribution. Table 4 presents the descriptive statistics for the residuals of the GJR-GARCH skew-t model. According to Ljung-Box and ARCH tests, there is no leftover autocorrelation in the mean or squared residuals. The p-values of the Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) tests for skew-t distribution with the estimated (ξ, ν) parameter values do not reject the null hypothesis, indicating the appropriateness of the distributional assumption. Finally, the Data Driven Smooth (DD) test for Uniformity, which checks whether the probability integral transforms of the residuals are uniformly distributed, does not reject the null either. In other words, the marginals have been modeled correctly and the resulting uniformly distributed data can be used in the copula estimation step.

Table 3: Estimation results for the marginals.

	μ_i	ω_i	α_i	β_i	γ_i	ξ_i	ν_i
S&P 500	0.029	0.017	0.002	0.894	0.18	0.893	7.052
	(0.008)	(0.003)	(0.008)	(0.012)	(0.021)	(0.013)	(0.568)
WTI	0.012	0.064	0.051	0.918	0.04	0.922	6.284
	(0.021)	(0.027)	(0.013)	(0.018)	(0.013)	(0.014)	(0.494)

Estimated parameters and standard errors (in parenthesis) for the GJR-GARCH model with skew-t errors for S&P 500 and WTI data.

Table 4: Descriptive statistics for GJR-GARCH model residuals.

	mean	Q2	sd	skew	kurtosis	min	max	LB(10)	ARCH(10)	KS	AD	DD
S&P 500	0.00	0.04	1.00	-0.57	5.42	-7.17	6.22	0.07	0.71	0.09	0.12	0.97
WTI	0.00	0.02	1.01	-0.42	6.22	-8.63	6.22	0.56	0.29	0.57	0.69	0.95

Descriptive statistics for GJR-GARCH skew-t model residuals. The LB(k) reports the p-value for the Ljung-Box test for autocorrelation of k lags, and ARCH(l) reports the p-value for the test for ARCH effects with l lags. KS and AD are the p-values for Kolmogorov-Smirnov and Anderson-Darling tests for skew-t distribution. Finally, DD is the p-value for the Data Driven Smooth test for Uniformity that checks whether the probability integral transforms of the residuals are uniformly distributed.

G Robustness checks

This section contains the results of the numerous robustness checks. In particular, we consider two extensions of the DSC-t copula model, namely, a mixture of t copulas, as well as time-varying degrees of freedom parameter. We also include two replications of the Empirical Illustration Section considering five different data sets, and also alternative factor identification strategies.

Mixture of student copulas

The proposed copulas in Section 2.2 of the main manuscript can only capture the symmetric positive or negative dependence at a point in time using the elliptical copulas or equally-weighted rotations of Archimedean copulas. To overcome this shortcoming, the weight of mixture copulas can also be estimated as a parameter in the model. In line with Loaiza-Maya et al. (2018), we investigate the performance of a five-parameter mixture Student-t copula, defined below, as it can provide insights into both directional dependence and asymmetry.

$$
\begin{aligned}
(u_{1,t}, u_{2,t}) \sim & c^{MIX}(u_{1,t}, u_{2,t}; w, \theta_{1t}, \nu_1, \theta_{2t}, \nu_2), \\
c^{MIX}(u_{1,t}, u_{2,t}; w, \theta_{1t}, \nu_1, \theta_{2t}, \nu_2) = & w c^T(u_{1,t}, u_{2,t}; \theta_{1t}, \nu_1) + \\
& (1-w) c^T(1-u_{1,t}, u_{2,t}; \theta_{2t}, \nu_2) \\
\theta_{it} = & \Lambda(\lambda_{it}) > 0, \text{for } i = 1, 2, \\
\lambda_{it} = & \lambda_{i0}(1-\beta) + \beta\lambda_{i,t-1} + \sigma_e e_t, \quad e_{it} \sim N(0,1).
\end{aligned} \tag{2}
$$

In this mixture, the weighting constant w is a parameter itself and the copula decides the direction of the asymmetry and fat-tailedness. Note that the correlation parameter θ is restricted to be positive for identification purposes. For $w > 0$ at any time point, the first mixture component captures positive dependence, meanwhile, the second component is a corresponding survival copula and captures a negative dependence structure. We fit the mixture copula to our data used in the main manuscript, the estimation results are in Table 5. We find that almost all weight is assigned to the first component, indicating the predominantly positive dependence structure between stock-oil log returns. The value of the log marginal likelihood is smaller as compared to the less complex DSC model with t copula. This can happen as a result of the smooth changes of the stock-oil dependence from positive to negative and vice versa, while the mixture copulas in Loaiza-Maya et al. (2018) aims to model abrupt changes at any time that are suitable for the temporal dependence of the return series.

Time varying degrees of freedom Student copulas

The average estimated degrees of freedom parameter in the DSC-t model is 17, with 10-90% credible intervals between 13 and 22. Nonetheless, it is likely that for most of the time points the stock-oil co-dependence has low tail dependence, and then in a particular stretch of time, such as crisis periods, for example, the dependence becomes high. Therefore, we wish to investigate whether the degrees of freedom parameter varies over time, as in Hansen (1994); Brooks et al. (2005); Creal et al. (2008), for example. We define a random walk process for the degrees of freedom parameter ν,

$$
\begin{aligned}
(u_{1,t}, u_{2,t}) &\sim c(u_{1,t}, u_{2,t}; \theta_t, \nu_t), \\
\theta_t &= \Lambda(\lambda_t), \\
\lambda_t &= \lambda_0(1-\beta) + \beta\lambda_{t-1} + \sigma_e e_t, \quad e_t \sim N(0,1), \\
\nu_t &= \nu_{t-1} + \sigma_\nu \eta_t, \quad \eta_t \sim N(0,1).
\end{aligned} \tag{3}
$$

We fit the DSC-t model with the time-varying degrees of freedom to our data used in the main manuscript, and, as seen from Table 5, the value of the log marginal likelihood is the same as in the DSC-t model with static degrees of freedom parameter. Note that the estimated parameter ν_1 is the initial value of the random walk process. The left panel of Figure 2 draws the filtered path of the parameter ν_t with 10-90% credible intervals in blue. We see that even though the parameter fluctuates over time, the changes are not dramatic, i.e. the evidence obtained from the data supports smooth and stable evolution of the degrees of freedom parameter. The overall level is almost always above 20, a finding in line with the estimated static ν in the DSC model. The right panel of Figure 2 draws a histogram of the average ν values at each time t. We can observe that there are some periods with lower degrees of freedom values (the long left tail), with the majority of values being around 25, indicating almost normal tails.

Table 5: Estimation results for the DSC model, a mixture of Student copulas, and DSC model with time-varying degrees of freedom.

	θ_{01}	θ_{02}	β	σ_e	σ_ν (or w)	ν_1	ν_2	LML
DSC Student	0.184		0.997	0.027		17.278		253.042
	(0.082;0.289)		(0.995;0.999)	(0.021;0.035)		(13.176;21.928)		
DSC MIX Student	0.050	0.220	0.999	0.101	0.998	18.306	8.856	246.319
	(0.009;0.120)	(0.020;0.607)	(0.999;1.000)	(0.076;0.127)	(0.994;1.000)	(13.631;23.462)	(2.809;18.588)	
DSC TVP Student	0.178		0.996	0.032	0.065	16.842		253.337
	(0.099;0.257)		(0.994;0.998)	(0.024;0.040)	(0.015;0.165)	(7.280;28.379)		

The table reports the estimation results for the DSC model with Student copulas. The numbers in the brackets show the [10% - 90%] credible intervals. We report the θ_0 equivalent value of λ_0 for ease of comparison. The fifth column of the table reports the weight in the DSC MIX Student copula model and the variance parameter of the random walk process in the time-varying Student copula model.

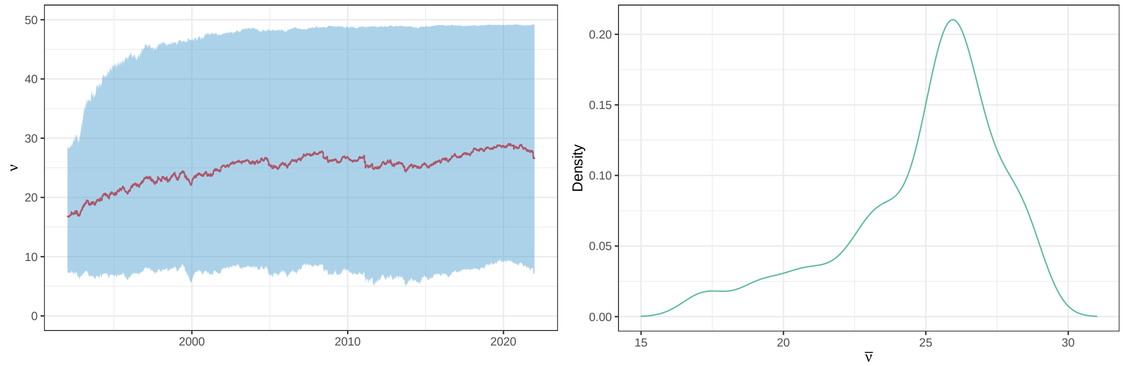

Figure 2: Time varying degrees of freedom parameter estimated from DSC Student copula model. The left panel shows the time varying degrees of freedom parameter estimated from DSC Student copula model with the 80% credible intervals during the period from 01/01/1990 to 31/12/2021. The right panel plot shows the density of its posterior means.

Stock-oil co-dependence for different sectors

As the Energy sector forms up to 5.1% of the S&P 500 index according to the `www.spglobal.com`, we perform a robustness study where we investigate the stock-oil relationship not on the S&P 500 as a whole, but using the five largest sectors in the S&P 500 such as Information Technology (S5INFT), Health Care (S5HLTH), Consumer Discretionary (S5COND), Communication Service (S5TELS), Financials (S5FINL). Note that we employ same the liquidity factor as for stock-oil returns in the main manuscript, as it is a proxy for the overall liquidity in the financial markets. Table 6 shows that the sign effects of the macro are consistent regardless of sector returns. Hence we can conclude that inflation/interest rate, uncertainty and liquidity factors are the main drivers of the long-run co-dependence between stock and oil returns.

Alternative factor identification

As a robustness check, we have considered an alternative approach, a purely-statistical one, for factor identification. In particular, we have analyzed all macroeconomic/financial variables at once and performed the PC analysis, where, using the screeplot, we have identified three principal components. Then, we have implemented the varimax and promax rotations with the goal of assigning one variable at most one factor, see Table 7 for the results. In both rotations, we can see that the varimax and promax variable assignments are strikingly similar to the ones obtained using economic reasoning.

The first PC represents the state of the economy in a wide sense, since, as opposed to the "original" factors, the first PC also contains the interest rate and spread. However, the direction of the effect is not so clear, as the first PC is negatively related to positive economic indicators, such as IP, CEAI and NAI, and at the same time the sign of the interest rate and spread are the opposite of what we would expect if the first PC was just a "negative economic indicator". Next,

Table 6: Estimation results for the DSM copula model with one macro-finance factor for different sectors in S&P 500 from 01/01/1990 till 31/12/2021 ($n = 8059$).

	λ_0	β	σ_e^2	δ	ω_2	ν	LML
(a) S5INFT - S&P 500 Information Technology							
DSM INR	0.161	0.994	0.033	-0.101	16.348	22.339	156.049
	(0.071;0.245)	(0.991;0.997)	(0.023;0.044)	(-0.193;0.001)	(4.951;27.458)	(16.121;30.110)	
DSM UNC	0.158	0.989	0.040	0.207	14.264	22.097	159.706
	(0.098;0.216)	(0.982;0.995)	(0.024;0.055)	(0.149;0.265)	(2.123;26.900)	(15.576;30.235)	
DSM LIQ	0.067	0.996	0.029	0.082	14.381	22.137	157.911
	(-0.072;0.220)	(0.994;0.998)	(0.022;0.038)	(0.006;0.155)	(4.134;25.558)	(15.596;29.503)	
DSM SOE	0.180	0.998	0.023	0.030	14.389	19.047	159.261
	(0.005;0.350)	(0.997;0.999)	(0.017;0.031)	(-0.133;0.205)	(3.919;25.316)	(14.260;24.959)	
(b) S5HLTH - S&P 500 Health Care							
DSM INR	0.060	0.992	0.038	-0.123	19.354	30.582	110.396
	(-0.008;0.127)	(0.987;0.995)	(0.029;0.048)	(-0.187;-0.060)	(8.077;28.739)	(20.016;44.425)	
DSM UNC	0.085	0.974	0.059	0.246	2.186	31.592	116.689
	(0.045;0.125)	(0.959;0.986)	(0.040;0.079)	(0.195;0.296)	(1.083;3.038)	(20.608;44.612)	
DSM LIQ	0.046	0.989	0.049	0.081	14.400	33.138	107.475
	(-0.024;0.112)	(0.984;0.993)	(0.039;0.060)	(0.026;0.133)	(2.877;26.651)	(21.251;47.563)	
DSM SOE	0.054	0.991	0.041	-0.044	11.750	27.680	107.846
	(-0.026;0.124)	(0.985;0.996)	(0.031;0.053)	(-0.128;0.040)	(1.470;26.404)	(18.623;39.636)	
(c) S5COND - S&P 500 Consumer Discretionary							
DSM INR	0.090	0.987	0.052	-0.146	14.577	19.918	163.372
	(0.019;0.157)	(0.980;0.994)	(0.037;0.069)	(-0.205;-0.087)	(3.985;27.916)	(14.235;26.693)	
DSM UNC	0.118	0.984	0.049	0.259	3.606	19.113	171.239
	(0.065;0.169)	(0.973;0.993)	(0.033;0.067)	(0.198;0.322)	(1.331;6.545)	(14.105;25.008)	
DSM LIQ	0.103	0.991	0.046	0.090	14.847	20.274	161.900
	(0.022;0.187)	(0.985;0.997)	(0.029;0.063)	(0.015;0.163)	(3.695;27.252)	(14.015;28.065)	
DSM SOE	0.121	0.986	0.057	-0.079	8.912	21.579	163.431
	(0.055;0.187)	(0.980;0.992)	(0.043;0.072)	(-0.142;-0.020)	(1.300;23.622)	(15.273;29.875)	
(d) S5TELS - S&P 500 Communication Services							
DSM INR	0.072	0.991	0.032	-0.096	15.959	23.502	80.251
	(0.017;0.126)	(0.987;0.994)	(0.025;0.040)	(-0.146;-0.048)	(5.345;26.326)	(16.440;31.918)	
DSM UNC	0.071	0.989	0.027	0.153	18.061	22.057	88.113
	(0.025;0.115)	(0.982;0.995)	(0.017;0.039)	(0.117;0.189)	(6.082;27.923)	(16.059;30.487)	
DSM LIQ	0.066	0.992	0.028	0.089	12.033	19.170	80.387
	(0.016;0.112)	(0.989;0.995)	(0.020;0.037)	(0.052;0.127)	(2.058;24.586)	(14.249;25.083)	
DSM SOE	0.129	0.995	0.029	-0.053	10.670	21.282	73.777
	(0.027;0.241)	(0.991;0.998)	(0.020;0.038)	(-0.158;0.074)	(1.486;24.290)	(14.972;28.609)	
(e) S5FINL - S&P 500 Financials							
DSM INR	0.144	0.996	0.025	-0.091	11.530	17.336	202.451
	(0.062;0.226)	(0.995;0.998)	(0.020;0.031)	(-0.168;-0.016)	(2.136;23.588)	(13.292;21.941)	
DSM UNC	0.136	0.988	0.041	0.264	4.321	20.955	203.237
	(0.082;0.187)	(0.982;0.993)	(0.030;0.052)	(0.207;0.323)	(1.388;10.629)	(14.786;28.049)	
DSM LIQ	0.033	0.996	0.028	0.140	12.937	18.756	196.076
	(-0.041;0.169)	(0.994;0.998)	(0.022;0.036)	(0.051;0.193)	(2.967;24.077)	(13.932;24.464)	
DSM SOE	0.149	0.995	0.034	-0.067	14.398	19.661	193.276
	(0.026;0.285)	(0.993;0.998)	(0.027;0.043)	(-0.148;0.016)	(3.420;26.911)	(14.611;25.861)	

The table reports the estimation results of the DSM copula model (with t copula). The long term dependence component is modelled using one explanatory variable with the restricted beta weighting function. The selected lag length in the weighting function is such that the maximum likelihood becomes insensitive to the its choice ($K = 24$). The numbers in the brackets show the [10% - 90%] credible intervals.

Table 7: Varimax and promax rotations for PCA on all variables with 3 principal components.

	Varimax			Promax		
	PC1	PC2	PC3	PC1	PC2	PC3
INF		-0.26			-0.25	
IR_EFFR	-0.29	-0.32		-0.29	-0.32	
T10Y3M	0.35			0.35		0.14
EPU	0.28		-0.31	0.29		-0.26
EMV			-0.28	0.11		-0.26
NFCIm	0.15		-0.34	0.16	-0.12	-0.31
LIXm_wti	0.11	0.42		0.10	0.42	
Amihm_wti		-0.31			-0.31	
Amivm_wti		0.30			0.30	
LIXm_sp500		0.44	0.12		0.45	0.10
Amihm_sp500		-0.47			-0.47	
Amivm_sp500		0.17			0.17	
IPa	-0.44			-0.44		
CEAIa	-0.41			-0.41		
NAIa	-0.48			-0.49		
CCI			0.46			0.47
BCI	0.20		0.44	0.19		0.48
CLI	0.13		0.51	0.11		0.53

the second principal component is mostly related to the liquidity measures, and its interpretation, in this case, is pretty straightforward: larger values of this PC indicate larger liquidity. This factor also contains the inflation and interest rate, both with negative signs, an indication of favorable economic/financial conditions. Finally, the third principal component could be regarded as the (un)certainty factor. It is negatively related to the three uncertainty measures, such as EPU, EMV and NFCI and positively related to the three confidence indices, which represent the opposite of uncertainty. Table 8 presents the correlations between all individual variables and the PCs obtained from the varimax rotation.

After we have constructed the principal components based on the varimax rotations, we have re-estimated the DSM copula model using the three factors. Table 9 reports the estimation results. The top panel contains the results for the three factors as a single explanatory variable, meanwhile, the bottom panel contains results for realized correlation-factor pairings. As seen from the table (both panels), the first PC has a positive and statistically significant effect on the stock-oil co-dependence. However, the economic interpretation is not so straightforward, as the economic meaning of the first factor seems rather ambiguous. The second factor represents (mostly) liquidity, and its effect on the co-dependence is positive yet insignificant, same as the liquidity factor in the main empirical application. Finally, the third factor could be seen as the certainty indicator, and it has a negative and statistically significant effect. The sign is the opposite to the uncertainty factor in the main empirical application, as expected.

Table 8: Correlation matrix of individual marcoeconomic/financial variables.

	Rcor	PC1_vx	PC2_vx	PC3_vx
INF	-0.192	-0.022	-0.447	-0.123
IR_EFFR	-0.484	-0.653	-0.614	-0.028
T10Y3M	0.089	0.635	-0.094	-0.147
EPU	0.405	0.572	0.091	0.415
EMV	0.217	0.232	0.112	0.413
NFCIm	0.179	0.323	-0.162	0.478
LIXm_wti	0.473	0.322	0.789	0.075
Amihm_wti	-0.122	0.039	-0.571	-0.070
Amivm_wti	0.265	0.209	0.570	0.058
LIXm_sp500	0.110	-0.035	0.799	-0.202
Amihm_sp500	-0.279	0.036	-0.856	0.112
Amivm_sp500	0.023	-0.104	0.301	-0.090
IPa	-0.329	-0.837	-0.113	0.041
CEAIa	-0.261	-0.775	-0.114	0.062
NAIa	-0.361	-0.898	0.079	0.022
CCI	0.060	0.076	0.048	-0.681
BCI	0.068	0.333	-0.035	-0.688
CLI	0.092	0.185	0.019	-0.772
PC1_vx	0.407	1.000	-0.000	0.000
PC2_vx	0.318	-0.000	1.000	-0.000
PC3_vx	0.102	0.000	-0.000	1.000

The first three principal components obtained from varimax rotations and the monthly realized correlation (Rcor) for 01/01/1990 till 31/12/2021 ($n = 8059$).

Table 9: Estimation results for the DSM copula model with one/two explanatory variables.

	λ_0	β	σ_e	δ_1	ω_1	δ_2	ω_2	ν	LML
DSM PC1 VX	0.224	0.993	0.038	0.147	9.113			20.240	249.910
	(0.148;0.303)	(0.990;0.996)	(0.028;0.048)	(0.042;0.253)	(1.216;23.578)			(14.531;27.309)	
DSM PC2 VX	0.183	0.997	0.028	0.108	15.802			17.817	253.790
	(0.072;0.289)	(0.995;0.998)	(0.021;0.035)	(-0.038;0.241)	(4.475;26.784)			(13.497;22.756)	
DSM PC3 VX	0.247	0.995	0.032	-0.435	5.937			18.963	254.034
	(0.155;0.339)	(0.991;0.998)	(0.024;0.042)	(-0.671;-0.155)	(1.183;22.330)			(14.062;24.488)	
DSM RCor - PC1 VX	0.089	0.972	0.064	1.099	4.239	0.070	15.352	20.788	257.343
	(0.032;0.144)	(0.957;0.985)	(0.046;0.083)	(0.856;1.366)	(2.141;6.385)	(0.018;0.122)	(4.848;25.746)	(14.987;27.648)	
DSM RCor - PC2 VX	0.129	0.982	0.048	0.835	6.164	0.095	14.816	19.107	257.625
	(0.060;0.209)	(0.968;0.993)	(0.033;0.067)	(0.342;1.270)	(1.801;14.819)	(-0.010;0.219)	(4.621;26.078)	(14.218;24.769)	
DSM RCor - PC3 VX	0.114	0.972	0.059	0.877	4.544	-0.240	3.555	19.388	257.116
	(0.060;0.172)	(0.950;0.990)	(0.036;0.086)	(0.512;1.193)	(1.724;7.464)	(-0.390;-0.099)	(1.183;7.292)	(14.101;25.706)	

The table reports the estimation results of the DSM copula model (with t copula). The long term dependence component is modelled using one/two explanatory variables with the restricted beta weighting function. Top panel contains results for one PC Varimax factor, and bottom panel contains results for the RCor plus one PC Varimax factor as explanatory variables. The selected lag length in the weighting function is such that the maximum likelihood becomes insensitive to the its choice ($K = 24$). The numbers in the brackets show the [10% - 90%] credible intervals.

To sum up, the second robustness check provides three important results. First, the resulting groups are strikingly similar to the ones formed using economic reasoning, however, in some cases, the economic interpretability is lost. Secondly, for those factors that maintain a clear economic meaning the sign and size of the effect are comparable to the main empirical application. Finally, in terms of log-marginal likelihood, the models that use economic factors generally present better in-sample fit.

H Variance and covariance forecasts

We estimate the GJR-GARCH model for daily log returns $r_{i,t}$, where $i = \{1 \text{ for S\&P 500}, 2 \text{ for WTI}\}$, and obtain the set of marginal parameters $\widehat{\Theta}_i = (\widehat{\mu}_i, \widehat{\omega}_i, \widehat{\alpha}_i, \widehat{\beta}_i, \widehat{\gamma}_i, \widehat{\xi}_i, \widehat{\nu}_i)$. Define the realized cumulative returns and the realized cumulative covariance of stock and oil during the k periods at time t:

$$
\begin{aligned}
r_{i,t+1:t+k} &= \sum_{j=1}^{k} r_{i,t+j}, \\
\text{RCov}_{t+1:t+k} &= \sum_{j=1}^{k} r_{1,t+j} r_{2,t+j}.
\end{aligned}
$$

Following Engle (2009), the k-step-ahead forecast of the cumulative covariance can be approximated by

$$
\text{Cov}(r_{1,t+1:t+k}, r_{2,t+1:t+k}) \approx \sum_{j=1}^{k} \rho_{t+j|t} \sqrt{\sigma^2_{1,t+j|t} \sigma^2_{2,t+j|t}}.
$$

The k-step-ahead forecasts for the variance at time t of the GJR-GARCH model are derived as:

$$
\begin{aligned}
\sigma^2_{i,t+1|t} &= E_t(\sigma^2_{i,t+1}) = \omega + (\alpha + \gamma \mathbf{1}\{\varepsilon_{i,t} < 0\})\varepsilon^2_{i,t} + \beta \sigma^2_{i,t}, \\
\sigma^2_{i,t+2|t} &= E_t(\sigma^2_{i,t+2}) = \omega + (\alpha + 0.5\gamma + \beta)\sigma^2_{i,t+1|t} \\
&= \frac{\omega}{1-\delta} + \delta(\sigma^2_{i,t+1|t} - \frac{\omega}{1-\delta}) \text{ where } \delta = \alpha + 0.5\gamma + \beta, \\
\sigma^2_{i,t+k|t} &= E_t(\sigma^2_{i,t+k}) = \frac{\omega}{1-\delta} + \delta^{k-1}(\sigma^2_{i,t+1|t} - \frac{\omega}{1-\delta}).
\end{aligned}
$$

The k-step-ahead forecast of the cumulative volatility forecast using the GJR-GARCH model at time t is $\sigma^2_{i,t+1:t+k|t}$:

$$
\begin{aligned}
\sigma^2_{i,t+1|t} &= E_t(\sigma^2_{i,t+1}) = \omega + (\alpha + \gamma \mathbf{1}\{\varepsilon_{i,t} < 0\})\varepsilon^2_{i,t} + \beta \sigma^2_{i,t}, \\
\sigma^2_{i,t+1:t+2|t} &= E_t(\sum_{j=1}^{2} \sigma^2_{i,t+j}) = \frac{2\omega}{1-\delta} + \frac{1-\delta^2}{1-\delta}(\sigma^2_{i,t+1|t} - \frac{\omega}{1-\delta}), \\
\sigma^2_{i,t+1:t+k|t} &= E_t(\sum_{j=1}^{k} \sigma^2_{i,t+j}) = \frac{k\omega}{1-\delta} + \frac{1-\delta^k}{1-\delta}(\sigma^2_{i,t+1|t} - \frac{\omega}{1-\delta}).
\end{aligned}
$$

For the k-step-ahead forecast of the correlation, we obtain $\rho_{t+j|t}$ as the equivalent Pearson correlation implied by copula models. At first, we obtain the k-step-ahead forecast of the stochastic process λ_t and use the transformation function $\Lambda_\rho = \sin(\frac{\pi}{2}\tau_\kappa) = \sin(\frac{\pi}{2}\frac{\exp(\lambda)-1}{\exp(\lambda)+1})$ to map the process into a restricted domain of the correlation:

$$\lambda_{t+k|t} = \lambda_\tau + (\lambda_t - \lambda_\tau)^k,$$
$$\rho_{t+k|t} = \Lambda_\rho(\lambda_{t+k|t}).$$

We compare the accuracy of covariance forecasts using the Mincer-Zarnowitz regression (Mincer and Zarnowitz, 1969) for the in-sample data. We regress the *ex-post* monthly realized covariance on a constant and the 22-days ahead cumulative covariance forecast. Table 10 reports the regression R^2 and the p-value of the F-test that the intercept is zero and the slope coefficient is one. The test does not reject the null at 5% confidence level for none of the models, and the R^2 is higher in DSM copula models with two factors than the simple DSC model. In other words, the DSM copula models produce 22-days ahead cumulative covariance forecasts that are closer to the actually observed realized covariance.

Table 10: Mincer-Zarnowitz Regressions for the in-sample data.

Model	R^2	p-value
DSC	0.292	0.077
DSM-RCor	0.262	0.374
DSM-UNC	0.398	0.433
DSM-RCor-UNC	0.384	0.656
DSM-INR-UNC	0.384	0.254
DSM-UNC-LIQ	0.397	0.956

The table reports the results for the Mincer-Zarnowitz regressions of monthly realized covariance on a constant and the 22-days ahead cumulative covariance forecast. The p-value is for the F-test that the intercept is zero and the slope coefficient is one.

References

Brooks, C., Burke, S. P., Heravi, S., and Persand, G. (2005). "Autoregressive conditional kurtosis." *Journal of Financial Econometrics*, 3(3): 399–421.

Colacito, R., Engle, R. F., and Ghysels, E. (2011). "A component model for dynamic correlations." *Journal of Econometrics*, 164(1): 45–59.

Conrad, C., Loch, K., and Rittler, D. (2014). "On the macroeconomic determinants of long-term volatilities and correlations in US stock and crude oil markets." *Journal of Empirical Finance*, 29(Dec): 26–40.

Creal, D., Koopman, S. J., and Lucas, A. (2008). "A General Framework for Observation Driven Time-Varying Parameter Models." *SSRN Electronic Journal*, 1–54.

Deligiannidis, G., Doucet, A., and Pitt, M. K. (2018). "The correlated pseudomarginal method." *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 80(5): 839–870.

Engle, R. (2002). "Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models." *Journal of Business & Economic Statistics*, 20(3): 339–350.

— (2009). *Anticipating correlations*. Princeton University Press.

Fernández, C. and Steel, M. F. (1998). "On Bayesian modeling of fat tails and skewness." *Journal of the American Statistical Association*, 93(441): 359–371.

Ferreira, J. T. and Steel, M. F. (2006). "A constructive representation of univariate skewed distributions." *Journal of the American Statistical Association*, 101(474): 823–829.

Ghalanos, A. (2022). "rugarch: Univariate GARCH models." R package version 1.4-7.

Hafner, C. M. and Manner, H. (2012). "Dynamic stochastic copula models: Estimation, inference and applications." *Journal of Applied Econometrics*, 27(2): 269–295.

Hansen, B. E. (1994). "Autoregressive Conditional Density Estimation." *International Economic Review*, 35(3): 705–730.

Hong, Y., Tu, J., and Zhou, G. (2007). "Asymmetries in stock returns: Statistical tests and economic evaluation." *The Review of Financial Studies*, 20(5): 1547–1581.

Joe, H. (2015). *Dependence Modeling with Copulas*. Boca Raton, FL: CRC Press.

Loaiza-Maya, R., Smith, M. S., and Maneesoonthorn, W. (2018). "Time series copulas for heteroskedastic data." *Journal of Applied Econometrics*, 33(3): 332–354.

Mincer, J. A. and Zarnowitz, V. (1969). "The evaluation of economic forecasts." In *Economic forecasts and expectations: Analysis of forecasting behavior and performance*, 3–46. NBER.

Tse, Y. K. and Tsui, A. K. C. (2002). "A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations." *Journal of Business & Economic Statistics*, 20(3): 351–362.

□