A dynamic leverage stochastic volatility model

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Abstract

Stock returns are considered as a convolution of two random processes that are the return innovation and the volatility innovation. The correlation of these two processes tends to be negative which is the so-called leverage effect. In this study, we propose a dynamic leverage stochastic volatility (DLSV) model where the correlation structure between the return innovation and the volatility innovation is assumed to follow a generalized autoregressive score (GAS) process. We find that the leverage effect is reinforced in the market downturn period and weakened in the market upturn period.

JEL-codes: C11, C52, C58

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1 Introduction

Stock returns are considered as a convolution of two random processes that are the volatility and the return innovation. The correlation of these two processes tends to be negative which is the so-called leverage effect. One explanation for this leverage effect is that the negative shock to the return may increase the financial leverage ratio between debt and equity. Therefore, the stock becomes riskier which results in a higher volatility. The leverage response between volatility and return news was initially analyzed by Black (1976) and further supported by the research of Christie (1982), Nelson (1991), and Harvey and Shephard (1996), among others. However, a majority of studies consider a constant leverage over time which is a rather restrictive assumption. On the other hand, time varying leverage has been documented in several studies, see Bandi and Renò (2012), Yu (2012), Bretó (2014), among others. The dynamic leverage not only helps to derive a better prediction of the volatility process but also can influence asymmetrically on the volatility smiles (Veraart and Veraart, 2012).

In this study, we propose a dynamic leverage stochastic volatility (DLSV) model where the correlation structure between the return innovation and the volatility innovation is assumed to follow a generalized autoregressive score (GAS) process; see Creal et al. (2013) and Harvey (2013) for an introduction to GAS. As the correlation is driven by an observation driven process, the number of latent parameters needed to be estimated is significantly smaller than that of a parameter driven process, hence the estimation is computationally less expensive. It is also straightforward to extend the current proposed inference algorithm for the fixed leverage stochastic volatility (LSV) model to a dynamic setting with a small marginal cost of computation. We find that the sign and magnitude of the temporal return innovations are the main factors that encourage the leverage effect. The leverage effect is strengthened in the market downturn period and is weakened in the market upturn period. The DLSV model improves both in-sample fit and out-of-sample forecast in comparison to the fixed leverage stochastic volatility (LSV) model.

The LSV model was first considered by Harvey and Shephard (1996). The model assumes a fixed intertemporal correlation between the lag return innovation and the volatility innovation. Jacquier et al. (2004) propose an alternative specification for the contemporaneous relation. Yu (2005) compares these two models and concludes that the model of Harvey and Shephard (1996) is a Euler approximation of the continuous time SV model which is also supported by empirical analysis. Recently, Catania (2020) introduces a generalization of these two models by adding more temporal return innovation lags in the volatility innovation equation. However, there is evidence that the relation between the return innovation and the volatility innovation is dynamic. Yu (2012) generalizes the correlation structure in the LSV model using a linear spline and finds a strong evidence against the fixed leverage effect. On the other hand, Bandi and Renò (2012) analyze the jump diffusion SV models and allow the leverage to depend on the state condition of the firm, e.g. the spot volatility, hence time varying. Veraart and Veraart (2012) propose a theoretical framework for the stochastic leverage in the LSV models. Bretó (2014) analyzes the S&P 500 return using a idiosyncratic stochastic leverage model and confirms the random walk LSV model outperforms the fixed LSV model. Chon and Kim (2020) propose a regime dependence leverage effect where the leverage can be different among volatility regimes.

We take a Bayesian approach for statistical inference in DLSV, and employ the Annealing Sequential Monte Carlo (ASMC) algorithm of Tran et al. (2014) and Duan and Fulop (2015) for posterior approximation. ASMC is an efficient sampler for Bayesian inference that combines the Anneal important sampling of Neal (2001) and the Sequential Monte Carlo method of Del Moral et al. (2006).

The rest of the paper is organized as follows. Section 2 introduces the DLSV model and its Bayesian inference is presented in Section 3. Section 4 illustrates the dynamic leverage effects of several stock market indexes, and Section 5 concludes. The Appendix contains some technical details.

2 Dynamic leverage stochastic volatility model

Let $\{y_t, t = 1, ..., T\}$ be the time series of financial returns. Harvey and Shephard (1996) propose the LSV model based on the SV model of Taylor (1986) as follows,

$$
\begin{aligned}
y_t &= \exp(0.5h_t)\epsilon_t, \\
h_t &= \mu_h(1-\phi) + \phi h_{t-1} + \sigma_\eta \eta_t, \qquad (1)\\
\eta_t &= \rho\epsilon_{t-1} + \sqrt{1-\rho^2}\zeta_t,
\end{aligned}
$$

where h_t is the log volatility of the financial return, ϵ_t is the return innovation and η_t is the volatility innovation which correlates with ϵ_{t-1}. The innovations ϵ_t and ζ_t are assumed to be mutually independent variables that follow standardized Normal distributions. The latent volatility process h_t is specified by the mean parameter μ_h, the persistence parameter $|\phi| < 1$, and the volatility parameter $\sigma_\eta > 0$. The leverage effect between shocks and volatility is captured through the dependence between ϵ_{t-1} and η_t in the sense that given $\rho < 0$, a negative shock to the returns is likely to result in an increase in the volatility. We extend Equation 1 and allow for the dynamic leverage effect by allowing ρ_t to follow a GAS process as follows,

$$\begin{aligned}
\eta_t &= \rho_t \epsilon_{t-1} + \sqrt{1-\rho_t^2}\zeta_t, \\
\rho_t &= \frac{\exp(f_t) - 1}{\exp(f_t) + 1}, \\
f_t &= \omega(1-b) + a s_{t-1} + b f_{t-1}, \\
s_{t-1} &= \frac{\partial \log p(y_{t-1}|f_{t-1})}{\partial f_{t-1}},
\end{aligned} \tag{2}$$

so that $Corr(\eta_t, \epsilon_{t-1}) = \rho_t$. We ensure the value of $\rho_t \in [-1, 1]$ by using a transformation function of a GAS process f_t. The process f_t is an observation driven process that is specified by the mean parameter ω, the persistence parameter b and an adjustment term calculated as a score function s_{t-1}. Here, we restrict $0 \leqslant b < 1$ for the stationary condition, see Creal et al. (2013). Appendix A shows the closed-form formula of the updated score. The updated score depends on both the sign and magnitude of the return innovations which leads to the change in the leverage effect. This model is different from the proposal of Bretó (2014) where f_t is a parameter driven process. One of the advantages is that our model reduces the number of latent parameters. Moreover, Blasques et al. (2015) show that the use of the score functions is robust to the model misspecification and leads to the minimization of the Kullback–Leibler divergence between the true conditional density and the model-implied conditional density. Koopman et al. (2016) provide empirical evidence that the GAS process is a good competitor for other parameter driven processes. We note that when $a = b = 0$, the model becomes a fixed LSV model. Also, when $a = b = \omega = 0$, we obtain the SV model without leverage effect.

3 Bayesian Estimation

Let's denote $\theta = (\mu_h, \phi, \sigma_\eta, \omega, a, b)$ be the set of fixed parameters in the DLSV model. We employ the ASMC algorithm of Tran et al. (2014) and Duan and Fulop (2015) to make Bayesian inference on the parameters of interest; see Appendix B for the details of ASMC. The ASMC also provides the marginal likelihood estimate, which is useful for model comparison.

We use vague but proper prior distributions for the parameters of the DLSV model. The priors are more informative in comparison to Kastner and Frühwirth-Schnatter (2014) to prevent the bad particles at the early stage of tempering. For the parameters in the volatility process, their priors are $\mu_h \sim \mathcal{N}(0,1)$, $\phi \sim$ Beta$(20, 1.5)$ and $\sigma_\eta^2 \sim$ Gamma$(\frac{1}{2}, \frac{1}{2B_\sigma})$ which is equivalent to $\pm\sqrt{\sigma_\eta^2} \sim \mathcal{N}(0, B_\sigma)$. The choice of prior for σ_η makes it less influential when the true value is small, see Kastner and Frühwirth-Schnatter (2014). For the parameters in the GAS process, we consider the following priors: $\omega \sim \mathcal{N}(0,2)$, $a \sim \mathcal{N}(0,1)\mathbf{I}_{\{a>0\}}$, $b \sim$ Beta$(30, 1)$. For the fixed LSV model, we assume that the fixed parameter $\rho \sim \mathbf{U}(-1, 1)$.

4 Empirical Illustration

This section illustrates the performance of the proposed DLSV model using three daily stock return indexes: the Dow Jones Industrial Average index (DJI), the Financial Times Stock Exchange 100 index (FTSE) and the Nikkei 225 stock index (NIKKEI). The data is obtained from Macrobond database during the period from 01/01/1990 to 31/12/2020. The returns are calculated as the first difference of the log of the stock index multiplied by 100. Table 1 describes their summary statistics. All the return series show some degree of heavier tails than the normal distribution and serial correlations.

Table 1: Summary statistics of the three stock returns (01/01/1990 - 31/12/2020)

	Mean	Std.	Skewness	Kurtosis	Minimum	Maximum	J-B test	Ljung-Box(12)	ARCH(12)
DJI	0.031	1.117	-0.402	12.946	-13.842	10.764	54.704***	109.846***	2263.322***
FTSE	0.013	1.116	-0.277	7.561	-11.512	9.384	18.773***	71.521***	1610.268***
NIKKEI	-0.005	1.500	-0.132	5.457	-12.111	13.235	9.476***	21.414***	1322.883***

Table reports the summary statistics of the three stock indexes. The returns are calculated as the log difference of the stock indexes multiplied by 100. We report the test statistics of the Jarque-Bera test for normality, the Ljung-Box test for the serial correlation and ARCH test for the GARCH effect. ***,**,* denote significant at 1%, 5%, 10% level.

Next, we divide the series into an in-sample period from 01/01/1990 to 31/12/2017 and an out-of-sample period from 01/01/2018 to 31/12/2020 . We demean each of the return series for the whole sample size and estimate the model parameters of the SV model without leverage, the LSV model and the proposed DLSV model using the in-sample datasets with the ASMC sampler. Table 2 reports the posterior mean estimates of the parameters. The credible interval of parameter a and b is significantly different from 0 which shows that the leverage effect is time-varying. The averages of the leverage effect are very similar between the LSV model and the DLSV model. However, the DLSV improves the marginal likelihood over the SV and the LSV model.

Table 2: Posterior estimates for the in-sample data (01/01/1990 - 31/12/2017)

Model	μ_h	ϕ	σ_η	ρ	a	b	LLH
(a) DJI							
SV	-0.387	0.976	0.044				-8973.318
	(-0.514;-0.258)	(0.972;0.981)	(0.037;0.051)				
LSV	-0.364	0.975	0.042	-0.650			-8852.364
	(-0.462;-0.265)	(0.971;0.979)	(0.036;0.049)	(-0.688;-0.609)			
DLSV	-0.371	0.977	0.039	-0.639	0.486	0.983	-8844.172
	(-0.456;-0.286)	(0.973;0.981)	(0.033;0.045)	(-0.695;-0.547)	(0.172;0.787)	(0.946;0.999)	
(b) FTSE							
SV	-0.231	0.980	0.029				-9468.208
	(-0.356;-0.106)	(0.976;0.984)	(0.025;0.034)				
LSV	-0.237	0.983	0.025	-0.685			-9359.464
	(-0.352;-0.123)	(0.980;0.986)	(0.021;0.029)	(-0.726;-0.640)			
DLSV	-0.246	0.985	0.024	-0.702	0.700	0.990	-9347.305
	(-0.354;-0.139)	(0.983;0.988)	(0.020;0.028)	(-0.750;-0.646)	(0.259;1.203)	(0.980;0.999)	
(c) NIKKEI							
SV	0.469	0.966	0.044				-11792.990
	(0.372;0.565)	(0.961;0.972)	(0.038;0.051)				
LSV	0.467	0.964	0.047	-0.563			-11692.408
	(0.392;0.543)	(0.958;0.969)	(0.040;0.054)	(-0.607;-0.519)			
DLSV	0.482	0.965	0.045	-0.565	0.516	0.973	-11686.520
	(0.395;0.565)	(0.959;0.969)	(0.039;0.052)	(-0.612;-0.510)	(0.141;0.950)	(0.932;0.997)	

Table reports the posterior means of the parameters of SV, LSV and DLSV models, together with the marginal likelihood estimates in the last column. The average leverage effect in the DLSV model is computed as $\rho = \frac{\exp(\omega)-1}{\exp(\omega)+1}$ for the ease of comparison. The ASMC sampler is run with $M = 1000$ particles. The numbers in the bracket show the [10% - 90%] credible intervals.

Figure 1 shows the filtered volatility and the filtered dynamic leverage effect. The patterns of the filtered volatility are similar among all series. Even though, the leverage effect of the NIKKEI series is weakest and it is more volatile at the end of the sample. On the other hand, the dynamic leverage effect of the DJI is quite stable through time while that of the FTSE is decreasing. We do not observe a strong relation between the change in the dynamic leverage effect and the level of the volatility which is similar to findings of Yu (2012) and Bretó (2014).

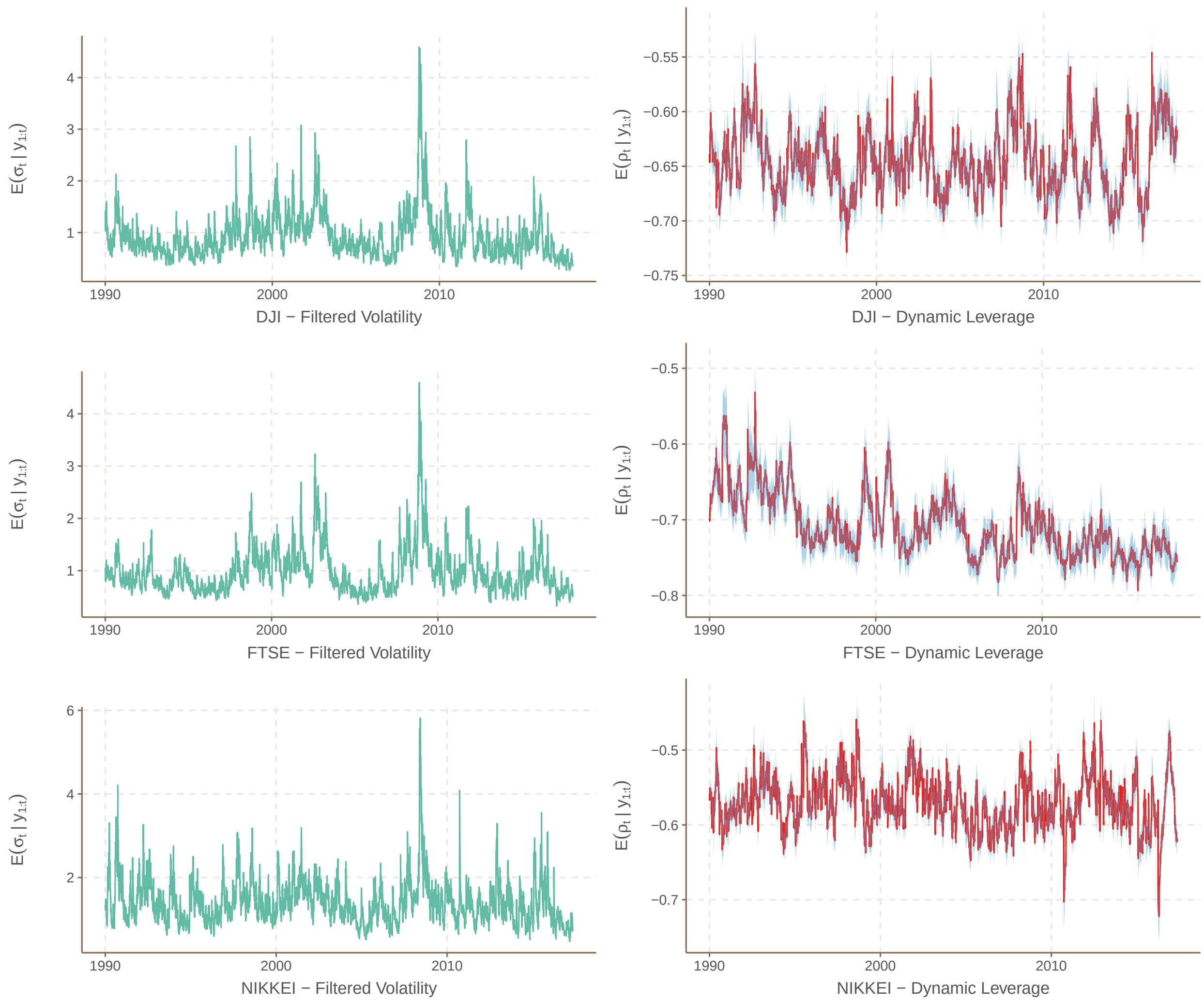

Figure 1: The filtered volatility and the filtered dynamic leverage effect.

The left hand side figure plots the filtered volatility and the right hand side figure plots the filtered dynamic leverage effect in red color with their 50% credible interval in the blue color. The results were obtained using a particle filter with 10000 particles.

We analyze the effect of the temporal return innovations $(\epsilon_t, \epsilon_{t-1})$ on the correlation ρ_{t+1}. Using Equation 2 and 3, we show the contour plot of the correlation ρ_{t+1} on the DJI temporal return innovations in Figure 2. Here, the GAS process f_t is assumed to stay at its long term mean ω and $\zeta_t = 0$. According to the sign and magnitude of $(\epsilon_t, \epsilon_{t-1})$, the leverage effect is reinforced when observing both strong temporal negative return shocks which is in contrast to when observing both strong temporal positive return shocks. The leverage is weaken during the uncertain market gain $(\epsilon_{t-1} > 0, \epsilon_t < 0)$ and it is strengthened during the uncertain market loss $(\epsilon_{t-1} < 0, \epsilon_t > 0)$.

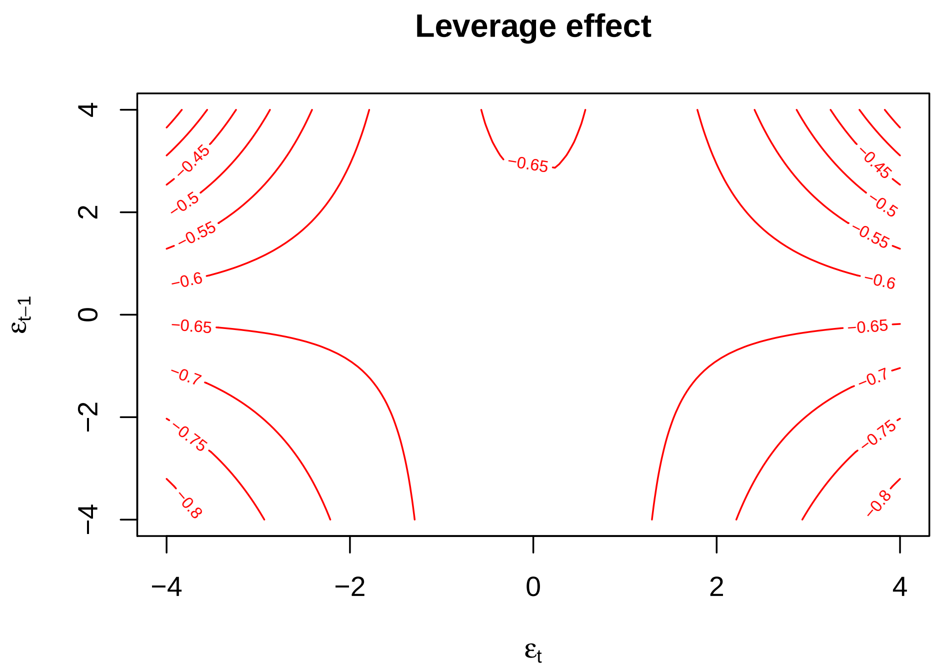

Figure 2: The contour plot shows the impact of the temporal return innovations $(\epsilon_t, \epsilon_{t-1})$ on the dynamic leverage correlation ρ_{t+1} for the DJI data. The current GAS process f_t is assumed to stay at its long term mean ω and $\zeta_t = 0$.

Table 3 compares the forecast of the competing models using the out-of-sample datasets. For predictive performance measurement, we employ the continuous ranked probability score (CRPS), the partial predicted score (PPS) and the quantile score (QS); see their detailed definition in Appendix C. The DLSV model is preferred overall which results in a higher CRPS, a higher PPS and a lower QS.

Table 3: Comparison of out-of-sample forecast (01/01/2018 - 31/12/2020)

	(a) DJI			(b) FTSE			(c) NIKKEI		
Model	CRPS	PPS	QS	CRPS	PPS	QS	CRPS	PPS	QS
SV	-0.6740	-1.4792	0.0612	-0.5858	-1.3997	0.0493	-0.6494	-1.5513	0.0452
LSV	-0.6691	-1.4571	0.0611	-0.5833	-1.3834	0.0470	-0.6430	-1.5146	0.0407
DLSV	-0.6689	-1.4560	0.0607	-0.5832	-1.3833	0.0472	-0.6430	-1.5133	0.0408

Table reports the CRPS, PPS and QS scores. The model with higher CRPS, higher PPS and lower QS is preferred.

5 Conclusion

In this study, we propose a dynamic LSV model where the correlation structure between the return innovation and the volatility innovation is assumed to follow a GAS process. Bayesian inference in DLSV is performed using the ASMC sampler of Tran et al. (2014) and Duan and Fulop (2015). We find that the DLSV model improves both in-sample fit and out-of-sample forecast in comparison to the fixed leverage effect LSV model and the standard SV model. Our key finding is that the leverage effect is reinforced in the market downturn period and is weakened in the market upturn period.

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Declarations of interest: none

Appendix

A Derivation of the score function

We apply the chain rule to derive the score function as,

$$
\begin{aligned}
s_t = \frac{\partial \log p(y_t|f_t)}{\partial f_t} &= \frac{\partial \log p(y_t|f_t)}{\partial h_t}\frac{\partial h_t}{\partial \rho_t}\frac{\partial \rho_t}{\partial f_t} \\
&= \frac{1}{2}(\epsilon_t^2 - 1)\sigma_\eta \left(\epsilon_{t-1} - \frac{\rho_t}{\sqrt{1-\rho_t^2}}\zeta_t\right)\frac{2\exp(f_t)}{(\exp(f_t)+1)^2}.
\end{aligned} \tag{3}
$$

B Annealing Sequential Monte Carlo algorithm

The ASMC algorithm combines the Anneal important sampling of Neal (2001) and the Sequential Monte Carlo method of Del Moral et al. (2006). The ASMC algorithm samples sequentially from the prior distribution $p(\theta)$ to the posterior distribution $p(\theta|y_{1:T})$ through a sequence of distributions that smoothly interpolate between the prior and the posterior distribution. The sequence of interpolation distributions is defined as,

$$\pi_i(\theta) := \pi_i(\theta|y_{1:T}) \propto \widehat{p}(y_{1:T}|\theta,u)^{\gamma_i} p(\theta),$$

where γ_i is referred as the level temperature for $i = 0,\dots,K$ and $0 = \gamma_0 < \gamma_1 < \dots < \gamma_K = 1$, $\widehat{p}(y_{1:T}|\theta,u)$ is the unbiased estimator of the integrated likelihood $p(y_{1:T}|\theta)$ and u is the set of the pseudo random numbers used in the particle filter to estimate the integrated likelihood $p(y_{1:T}|\theta)$. Algorithm 1 summarizes the main steps in the ASMC algorithm. We start to sample a set of M weighted particles $\{W_0^j, \theta_0^j\}_{j=1}^M$ from the prior distribution $p(\theta)$. At each iteration i of the ASMC algorithm, we reweight the set of M weighted particles $\{W_{i-1}^j, \theta_{i-1}^j\}_{j=1}^M$ obtained from the previous interpolation density $\pi_{i-1}(\theta)$ to approximate the target interpolation density $\pi_i(\theta)$. If the efficiency of these weighted particles measured by the effective sample size (ESS) is below a threshold, the particles are resampled. Then, in order to protect from weight degeneracy, the particles are refreshed by a Markov kernel whose invariant distribution is $\pi_i(\theta)$. We incorporate the Correlated Pseudo Marginal (CPM) approach by Deligiannidis et al. (2015) into the Markov move step to deviate from the unstable estimated integrated likelihood caused by particle filter, and use a random walk proposal for the new value of θ. The ASMC algorithm can be run in parallel for the particle moves in the Markov move step, so we can take advantage of the fast computation in a high-performance computing server. The ASMC algorithm also provides an estimate of log marginal likelihood. The pseudo-code implementation of ASMC is given in Algorithm 1.

C Predictive scores

The continuous ranked probability score (CRPS), partial predicted score (PPS) and the quantile score (QS) are defined as follows

$$\begin{aligned}
\text{CRPS} &= \frac{1}{T}\sum_{t=1}^{T}\hat{\sigma}_t\left[\frac{1}{\sqrt{\pi}} - 2\phi\left(\frac{y_t}{\hat{\sigma}_t}\right) - \frac{y_t}{\hat{\sigma}_t}\left(2\Phi\left(\frac{y_t}{\hat{\sigma}_t}\right) - 1\right)\right], \\
\text{PPS} &= \frac{1}{T}\sum_{t=1}^{T}\log p(y_t|y_{0:t-1}, \widehat{\theta}), \\
\text{QS} &= \frac{1}{T}\sum_{t=1}^{T}(\alpha - \mathbf{I}_{y_t<q_{t,\alpha}})(y_t - q_{t,\alpha}),
\end{aligned}$$

where ϕ and Φ denote the probability density function and the cumulative distribution function of a standard Normal variable, T is the number of out-of-sample periods, $\hat{\sigma}_t$ is the mean predicted volatility, $\widehat{\theta}$ is the posterior mean estimate of the model parameters, and $q_{t,\alpha}$ is the quantile forecast of y_t given $y_{0:t-1}$ at the probability level $\alpha = 0.1$. The interested reader is refered to Gneiting and Raftery (2007) and Taylor (2019) for more details.

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Algorithm 1 The ASMC for the DLSV model

1. Sample $\theta_0^j \sim p(\theta)$, $u_0^j \sim p(u)$ and set $W_0^j = 1/M$ for $j = 1...M$
2. **For** $i = 1, ..., K$,

Step 1: **Reweighting**: Compute the unnormalized weights

$$w_i^j = W_{i-1}^j \frac{\widehat{p}(y_{1:T}|\theta_{i-1}^j, u_{i-1}^j)^{\gamma_t} p(\theta_{i-1}^j)}{\widehat{p}(y_{1:T}|\theta_{i-1}^j, u_{i-1}^j)^{\gamma_{i-1}} p(\theta_{i-1}^j)} = W_{i-1}^j \widehat{p}(y_{1:T}|\theta_{i-1}^j, u_{i-1}^j)^{\gamma_i - \gamma_{i-1}}, \quad j = 1, ..., M \tag{4}$$

and set the new normalized weights

$$W_i^j = \frac{w_i^j}{\sum_{s=1}^M w_i^s}, \quad j = 1, ..., M. \tag{5}$$

Step 2: Compute the effective sample size (ESS):

$$\text{ESS} = \frac{1}{\sum_{j=1}^M \left(W_i^j\right)^2}. \tag{6}$$

if ESS $< cM$ for some $0 < c < 1$, **then**

(i) **Resampling**: Resampling from $\{\theta_{i-1}^j, u_{i-1}^j\}_{j=1}^M$ using the weights $\{W_i^j\}_{j=1}^M$, and then set $W_i^j = 1/M$ for $j = 1...M$, to obtain the new equally-weighted particles $\{\theta_i^j, u_i^j, W_i^j\}_{j=1}^M$.

(ii) **Markov move**: Parallel for each $j = 1, ..., M$, move the samples θ_i^j, u_i^j for N_{CPM} CPM steps:

(a) Sample θ_i^{j*} from the random walk proposal density $q(\theta_i^{j*}|\theta_i^j)$.

(b) Sample $\epsilon^j \sim \mathbf{N}(0_D, I_D)$ and set $u_i^{j*} = \rho u_i^j + \sqrt{1-\rho^2}\epsilon^j$ with $\rho \in (-1,1)$ is a correlation factor.

(c) Compute the estimated likelihood $\widehat{p}(y_{1:T}|\theta_i^{j*}, u_i^{j*})$ using a particle filter

(d) Set $\theta_i^j = \theta_i^{j*}$ and $u_i^j = u_i^{j*}$ with the probability

$$\min\left(1, \frac{\widehat{p}(y_{1:T}|\theta_i^{j*}, u_i^{j*})^{\gamma_i} p(\theta_i^{j*})}{\widehat{p}(y_{1:T}|\theta_i^j, u_i^j)^{\gamma_i} p(\theta_i^j)} \frac{q(\theta_i^j|\theta_i^{j*})}{q(\theta_i^{j*}|\theta_i^j)}\right), \tag{7}$$

otherwise keep θ_i^j, u_i^j unchanged.

end

3. The log of marginal likelihood estimate is

$$\log\widehat{p}(y_{1:T}) = \sum_{i=1}^K \log\left(\sum_{j=1}^M w_i^j\right). \tag{8}$$

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