Parallel Bayesian inference for high dimensional dynamic factor copulas

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Online Appendix

A Monte Carlo Study

In this section, we illustrate the proposed Bayesian methodology using simulated data from the MGSt one factor copula model. We generate a random sample of $d = 100$ time series with $G = 10$ groups and a time length $T = 1000$ from the dynamic factor model (6-8). The numbers of time series in each groups are randomized and the parameters are fixed for the simulation purpose. More precisely, we set $a = 0.05$ and $b = 0.98$ as the parameter control for the dynamic correlation in each group, the degree of freedom ν is generated as a sequence from $\nu_1 = 6$ to $\nu_{10} = 24$ with equally space and similarly the skewness γ is generated as a sequence from $\gamma_1 = -0.5$ to $\gamma_{10} = 0.4$. The expected correlation, ρ_c ,between pseudo observation x_t and the latent factor z_t are generated in the range $[0.1, 0.9]$ with equally space, which results in values for f_{igc} ranging in the interval $[0.2, 3]$. The latent variable z_t is generated from a standard Gaussian distribution and ξ_{gt} is generated from $IG(\nu_g/2, \nu_g/2)$.

We repeat each simulation 100 times and record the posterior mean. We estimate the set of true parameters, ϑ, using $10,000$ MCMC iterations where the first $5,000$ are discarded as burn-in iterations. The algorithm seems to perform adequately and convergence is fast. Practically, all the posteriors reached convergence after 1000 iterations. We retain every 10-th iterations to reduce autocorrelation. In each simulation, the algorithm takes around 13 minutes, 35 minutes and 45 minutes for the Gaussian, Student-t and MGSt one factor copula model, respectively, on an Intel Core i7-4770 processor (4 cores - 8 threads - 3.4GHz), although we only choose to report for the MGSt factor copula model.

Table 1 shows some summary statistics of the posterior mean in each MCMC iterations. In general, most of the parameters are correctly estimated. When the degree of freedom increases, its posterior estimation also becomes less accurate which leads to higher standard deviation of both ν and γ. On the other hand, the posterior estimation of the correlation becomes more accurate when there is a higher correlation with the latent variable. Similarly, the number of the assets in

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the groups also contributes to the standard deviation of parameters. We have observed that there is negative correlation between MCMC samples of a and b which means that if the posterior mean of a overestimates its true value, the value of b will underestimate its true value. Normally, the posterior variance of z_t reduces when the dimension increases.

Table 1: Simulation results

	True	Bias	Std	Median	90%	10%	Diff (90%-10%)
a	0.050	0.007	0.015	0.055	0.075	0.042	0.033
b	0.980	-0.029	0.071	0.972	0.985	0.908	0.077
ν_1	6.000	0.062	0.277	6.045	6.402	5.770	0.632
ν_2	8.000	0.022	0.445	8.002	8.648	7.439	1.209
ν_3	10.000	0.041	0.695	9.960	10.828	9.235	1.593
ν_4	12.000	0.052	0.969	12.092	13.326	10.787	2.539
ν_5	14.000	-0.135	1.287	13.763	15.737	12.194	3.543
ν_6	16.000	-0.102	1.584	15.772	17.656	13.965	3.692
ν_7	18.000	-0.872	1.797	17.007	19.391	15.084	4.307
ν_8	20.000	-1.087	2.056	18.905	21.504	16.644	4.860
ν_9	22.000	-1.694	2.381	20.190	23.043	17.313	5.730
ν_{10}	24.000	-1.713	2.985	22.260	26.019	18.301	7.718
γ_1	-0.500	-0.003	0.024	-0.501	-0.471	-0.536	0.065
γ_2	-0.400	-0.000	0.025	-0.398	-0.372	-0.435	0.063
γ_3	-0.300	0.006	0.028	-0.293	-0.262	-0.323	0.061
γ_4	-0.200	0.003	0.032	-0.197	-0.160	-0.232	0.072
γ_5	-0.100	-0.002	0.028	-0.102	-0.068	-0.138	0.070
γ_6	0.000	0.002	0.033	-0.002	0.045	-0.036	0.081
γ_7	0.100	-0.003	0.034	0.100	0.139	0.052	0.087
γ_8	0.200	-0.008	0.044	0.196	0.246	0.138	0.108
γ_9	0.300	-0.025	0.040	0.274	0.330	0.220	0.110
γ_{10}	0.400	-0.036	0.056	0.365	0.424	0.295	0.128
ρ_1	0.100	0.022	0.058	0.119	0.188	0.059	0.129
ρ_{12}	0.189	0.001	0.060	0.195	0.255	0.130	0.125
ρ_{23}	0.278	-0.002	0.065	0.282	0.339	0.190	0.149
ρ_{34}	0.367	-0.006	0.046	0.361	0.417	0.312	0.105
ρ_{45}	0.456	-0.005	0.047	0.449	0.506	0.401	0.104
ρ_{56}	0.544	-0.005	0.047	0.544	0.589	0.487	0.103
ρ_{67}	0.633	-0.003	0.036	0.633	0.671	0.582	0.090
ρ_{78}	0.722	-0.008	0.042	0.720	0.749	0.679	0.071
ρ_{89}	0.811	-0.001	0.024	0.815	0.833	0.785	0.048
ρ_{100}	0.900	-0.005	0.025	0.900	0.912	0.881	0.031

Summary statistic for the posterior mean of the MGSt factor copula model in the Monte Carlo study. We choose some selected value of ρ_c to conserve space.

B Number of assets in each group sector

Table 2: Stock Groups

Group	SIC code	Industry	Number of Firms
1	1000 - 1999	Mining and Construction	13
2	2000 - 2099	Food and Beverage	7
3	2830 - 2839	Pharmacy	7
4	2800 - 2899	Plastic Material and Plant Chemical	7
5	2100 - 2999	Textile and Papers	12
6	3000 - 3599	Steels	17
7	3600 - 3799	Home Appliance and Automobile	15
8	3800 - 3999	Electronics	8
9	4000 - 4899	Transportation and Telecommunication	7
10	4900 - 5999	Retail and Distribution	11
11	6300 - 6399	Insurance Firms	18
12	6000 - 6999	Finance (not include Insurance)	18

The table shows the number of firms in each group sectors. We classify the categories based on the SIC code.

C Univariate marginal distribution

Stock ID	c_i	ϕ_{1i}	ω_i	α_{1i}	β_{1i}	γ_i	$\xi_{i\eta}$	$\nu_{i\eta}$	K-S	AD	Neyman
AAPL	0.170	0.011	0.028	-0.074	0.977	0.181	1.030	5.303	0.159	0.525	0.686
JPM	0.038	-0.066	0.010	-0.087	0.993	0.165	1.020	5.176	0.306	0.614	0.771
JNJ	0.031	-0.013	-0.011	-0.111	0.959	0.246	1.011	5.699	0.132	0.243	0.888
BAC	-0.012	-0.018	0.016	-0.074	0.992	0.222	1.046	4.397	0.215	0.479	0.986
XOM	0.025	-0.069	0.010	-0.078	0.980	0.180	0.952	6.216	0.284	0.499	0.992
WFC	0.036	-0.091	0.010	-0.082	0.994	0.201	1.038	4.733	0.295	0.540	0.982
INTC	0.054	-0.010	0.007	-0.042	0.993	0.104	0.993	4.869	0.313	0.614	0.899
CVX	0.037	-0.042	0.009	-0.080	0.986	0.151	0.871	9.759	0.487	0.769	0.901
CSCO	0.041	-0.006	0.002	-0.031	0.997	0.075	0.991	3.554	0.130	0.450	0.776
UNH	0.050	-0.019	0.009	-0.063	0.992	0.104	0.985	4.316	0.176	0.664	0.949
BA	0.031	0.021	0.013	-0.074	0.988	0.135	0.978	5.662	0.510	0.898	0.921
PG	0.009	-0.043	0.002	-0.089	0.985	0.147	0.976	3.815	0.390	0.560	0.812
C	-0.028	0.029	0.018	-0.068	0.991	0.204	1.021	4.492	0.219	0.406	0.970
DIS	0.076	-0.038	0.009	-0.082	0.989	0.124	1.015	5.474	0.279	0.682	0.757
IBM	0.043	-0.019	0.009	-0.081	0.980	0.169	1.003	4.557	0.490	0.942	0.991
NVDA	0.046	-0.023	0.007	-0.029	0.996	0.106	1.028	4.766	0.376	0.777	0.986
GE	0.007	0.007	0.011	-0.081	0.990	0.170	0.995	4.645	0.251	0.588	0.999
MCD	0.032	-0.039	0.001	-0.024	0.994	0.116	0.934	5.552	0.113	0.638	0.833
MO	0.049	-0.016	0.003	-0.088	0.975	0.125	0.895	5.610	0.504	0.606	0.994
HON	0.053	-0.023	0.011	-0.101	0.988	0.136	1.002	7.518	0.195	0.216	0.897
MDT	0.026	-0.033	0.009	-0.060	0.986	0.087	0.972	4.901	0.206	0.474	0.922
ABT	0.033	-0.005	0.007	-0.061	0.979	0.137	1.024	5.698	0.188	0.427	0.891
UTX	0.044	-0.019	0.008	-0.105	0.986	0.118	0.990	7.039	0.345	0.419	0.773
GILD	0.100	-0.045	0.033	-0.056	0.973	0.119	1.017	3.952	0.650	0.717	0.829
GS	0.045	-0.042	0.008	-0.056	0.994	0.120	1.011	5.152	0.414	0.532	0.690
MS	0.036	-0.012	0.019	-0.067	0.991	0.171	1.040	5.053	0.171	0.297	0.673
TXN	0.043	-0.059	0.003	-0.062	0.996	0.036	0.969	6.277	0.339	0.669	0.937
CAT	0.066	0.013	0.008	-0.062	0.994	0.127	1.013	5.077	0.476	0.743	0.813
LMT	0.057	-0.056	0.008	-0.049	0.986	0.122	0.966	6.237	0.160	0.595	0.928
QCOM	0.057	-0.003	0.011	-0.082	0.989	0.133	1.034	4.366	0.082	0.431	0.862
SLB	0.078	-0.037	0.009	-0.055	0.994	0.104	1.037	6.817	0.533	0.707	0.958
BLK	0.064	-0.032	0.013	-0.074	0.991	0.157	1.025	5.667	0.333	0.616	0.923
NKE	0.064	-0.031	0.008	-0.069	0.991	0.125	0.994	3.957	0.802	0.772	0.982
USB	0.012	-0.080	0.008	-0.073	0.994	0.189	0.979	5.821	0.552	0.749	0.737
AXP	0.067	-0.073	0.008	-0.063	0.994	0.163	0.988	5.343	0.177	0.300	0.757
PNC	0.028	-0.059	0.012	-0.066	0.992	0.203	1.011	4.721	0.085	0.389	0.869
SCHW	0.055	-0.075	0.015	-0.060	0.991	0.121	1.037	5.694	0.307	0.471	0.945
CB	0.037	-0.077	0.009	-0.072	0.988	0.210	0.964	5.303	0.361	0.712	0.958
COP	0.081	-0.050	0.012	-0.061	0.985	0.169	0.933	9.858	0.101	0.481	0.492
FDX	0.032	-0.043	0.006	-0.060	0.995	0.096	1.014	5.943	0.261	0.572	0.965
GD	0.051	-0.047	0.010	-0.080	0.987	0.107	1.005	5.927	0.184	0.268	0.845
MU	0.117	0.010	0.017	-0.051	0.993	0.109	1.039	5.747	0.064	0.527	0.901
AMAT	0.056	-0.042	0.012	-0.038	0.991	0.111	1.055	5.416	0.220	0.419	0.795
AMT	0.064	-0.072	0.010	-0.093	0.988	0.153	0.992	5.691	0.476	0.830	0.873
RTN	0.047	-0.030	0.011	-0.067	0.979	0.137	0.949	8.678	0.309	0.686	0.856
CL	0.044	-0.067	0.003	-0.049	0.984	0.132	1.025	4.509	0.143	0.350	0.999
NOC	0.062	-0.025	0.009	-0.076	0.984	0.125	0.927	6.715	0.368	0.891	0.864
EOG	0.100	-0.053	0.019	-0.063	0.987	0.143	1.008	7.945	0.686	0.922	0.804
AGN	0.127	-0.020	0.038	-0.067	0.960	0.184	1.087	4.345	0.074	0.627	0.879
MET	0.026	-0.050	0.015	-0.093	0.991	0.175	0.983	5.165	0.229	0.545	0.977
PRU	0.062	-0.043	0.013	-0.086	0.992	0.191	0.973	5.552	0.346	0.535	0.794
F	0.089	0.039	0.010	-0.028	0.994	0.162	1.061	4.699	0.360	0.685	0.787
OXY	0.060	-0.040	0.008	-0.038	0.994	0.140	0.967	8.151	0.184	0.448	0.940
CI	0.093	-0.089	0.013	-0.092	0.990	0.134	1.004	5.438	0.178	0.417	0.716
SO	0.028	-0.054	-0.002	-0.029	0.979	0.176	1.005	5.926	0.280	0.647	0.693
TRV	0.050	-0.127	0.006	-0.062	0.991	0.173	0.966	4.792	0.662	0.747	0.683
VLO	0.108	0.013	0.020	-0.042	0.988	0.136	0.994	6.802	0.072	0.609	0.526
BAX	0.033	-0.026	0.011	-0.081	0.980	0.126	0.982	4.440	0.199	0.557	0.804
CCL	0.009	-0.017	0.006	-0.056	0.996	0.111	1.010	4.723	0.283	0.359	0.925
ECL	0.059	-0.099	0.011	-0.066	0.980	0.187	0.997	5.578	0.459	0.838	0.855
KMB	0.034	-0.094	0.000	-0.028	0.989	0.122	0.964	4.581	0.668	0.933	0.909

TGT	0.029	-0.060	0.004	-0.044	0.996	0.113	1.036	4.620	0.316	0.626	0.842
FOXA	0.034	-0.040	0.012	-0.091	0.992	0.120	1.014	6.365	0.334	0.574	0.749
HUM	0.086	-0.064	0.011	-0.044	0.992	0.097	1.039	3.740	0.493	0.580	0.987
AFL	0.032	-0.085	0.009	-0.084	0.993	0.157	0.983	4.405	0.609	0.753	0.877
PGR	0.027	-0.076	0.006	-0.040	0.994	0.139	0.985	4.272	0.089	0.448	0.801
PSA	0.062	-0.102	0.003	-0.035	0.996	0.147	0.995	7.431	0.576	0.860	0.831
ALL	0.036	-0.084	0.008	-0.088	0.991	0.182	1.011	4.918	0.570	0.714	0.880
ADI	0.012	-0.045	0.006	-0.073	0.995	0.057	0.961	5.127	0.461	0.543	0.778
MCK	0.079	-0.056	0.012	-0.070	0.986	0.121	1.035	4.214	0.286	0.511	0.934
WDC	0.125	-0.038	0.009	-0.063	0.995	0.087	1.019	4.836	0.201	0.571	0.942
ALXN	0.113	-0.036	0.062	-0.107	0.962	0.172	1.077	4.896	0.257	0.576	0.593
APH	0.074	-0.024	0.006	-0.088	0.995	0.105	1.034	4.080	0.146	0.476	0.917
ADM	0.036	-0.055	0.010	-0.037	0.992	0.118	0.963	4.067	0.384	0.515	0.954
EW	0.108	-0.020	0.028	-0.050	0.980	0.109	1.003	2.660	0.255	0.543	0.789
FCX	0.016	0.013	0.007	-0.063	0.996	0.094	0.957	8.029	0.312	0.334	0.555
TROW	0.049	-0.098	0.008	-0.080	0.994	0.154	0.972	5.547	0.245	0.645	0.980
VFC	0.096	-0.020	0.010	-0.068	0.990	0.140	1.047	4.815	0.154	0.574	0.666
PXD	0.152	-0.004	0.022	-0.052	0.988	0.149	1.037	7.183	0.297	0.685	0.992
MYL	0.056	-0.001	0.018	-0.070	0.986	0.130	1.016	4.257	0.194	0.739	0.882
GLW	0.048	0.015	0.007	-0.052	0.995	0.099	1.007	4.162	0.274	0.708	0.768
K	0.033	-0.066	0.001	-0.044	0.976	0.153	0.986	3.861	0.485	0.748	0.773
ED	0.016	-0.051	-0.001	-0.039	0.977	0.160	0.924	6.106	0.466	0.589	0.988
IP	0.038	-0.049	0.009	-0.058	0.994	0.143	0.948	5.985	0.539	0.860	0.895
NUE	0.005	-0.063	0.006	-0.069	0.995	0.086	0.962	8.082	0.355	0.814	0.823
OKE	0.094	0.010	0.024	-0.107	0.970	0.189	0.983	4.623	0.159	0.573	0.834
TSN	0.086	-0.041	0.009	-0.041	0.994	0.089	0.966	4.118	0.697	0.822	0.913
DLR	0.070	-0.039	0.009	-0.042	0.994	0.151	0.959	5.909	0.185	0.331	0.785
BFB	0.038	-0.072	0.012	-0.085	0.985	0.126	0.997	4.434	0.180	0.317	0.663
CBS	0.059	-0.035	0.013	-0.090	0.992	0.164	1.034	5.751	0.711	0.944	0.909
CNC	0.103	-0.018	0.024	-0.066	0.987	0.081	1.035	3.545	0.328	0.592	0.910
KLAC	0.060	-0.044	0.001	-0.049	0.999	0.058	0.979	4.975	0.121	0.440	0.726
LEN	0.003	0.033	0.005	-0.058	0.998	0.112	1.107	6.799	0.335	0.612	0.752
MHK	0.054	0.032	0.011	-0.069	0.993	0.114	1.065	5.692	0.222	0.499	0.892
DOV	0.052	-0.011	0.011	-0.091	0.990	0.118	0.980	6.552	0.199	0.676	0.898
EXPE	0.058	0.012	0.005	-0.061	0.997	0.064	1.006	3.478	0.279	0.659	0.850
IDXX	0.084	-0.008	0.012	-0.047	0.988	0.146	0.972	4.097	0.282	0.551	0.880
L	0.023	-0.106	0.007	-0.080	0.992	0.169	0.945	5.112	0.198	0.312	0.890
TAP	0.042	-0.019	0.032	-0.087	0.964	0.164	1.022	3.653	0.260	0.488	0.870
ANDV	0.114	-0.019	0.016	-0.037	0.992	0.119	1.007	7.802	0.425	0.635	0.813
BLL	0.052	-0.055	0.006	-0.099	0.991	0.086	0.968	5.013	0.356	0.654	0.912
DHI	0.032	0.009	0.006	-0.028	0.997	0.114	1.109	5.665	0.261	0.682	0.980
EMN	0.071	-0.042	0.016	-0.069	0.989	0.155	1.015	4.264	0.382	0.630	0.965
HES	0.099	-0.041	0.008	-0.054	0.995	0.114	0.959	8.616	0.228	0.669	0.712
NOV	0.101	-0.031	0.006	-0.049	0.996	0.118	0.972	6.532	0.515	0.774	0.843
TPR	0.049	0.033	0.002	-0.046	0.998	0.101	0.999	4.089	0.161	0.456	0.639
XL	0.077	-0.066	0.007	-0.086	0.995	0.194	0.993	4.557	0.079	0.267	0.459
EQT	0.074	0.003	0.025	-0.085	0.982	0.156	0.963	6.285	0.050	0.164	0.930
JBHT	0.037	-0.047	0.005	-0.068	0.996	0.090	1.033	5.482	0.156	0.368	0.401
NBL	0.075	-0.040	0.016	-0.072	0.989	0.130	0.971	8.645	0.196	0.449	0.938
ARE	0.020	-0.048	0.007	-0.062	0.995	0.165	0.951	6.673	0.251	0.408	0.789
XRAY	0.025	-0.021	0.011	-0.089	0.986	0.143	0.969	4.543	0.286	0.511	0.809
IFF	0.060	-0.106	0.012	-0.073	0.983	0.146	0.994	4.259	0.567	0.427	0.885
MLM	0.046	0.022	0.010	-0.057	0.993	0.132	1.026	5.271	0.515	0.719	0.732
MKC	0.060	-0.065	0.005	-0.022	0.982	0.146	1.024	4.127	0.073	0.338	0.648
NWL	0.040	-0.010	0.005	-0.060	0.996	0.081	1.019	4.286	0.349	0.690	0.904
PVH	0.066	0.044	0.013	-0.082	0.991	0.133	1.041	5.742	0.899	0.889	0.856
CHD	0.050	-0.045	0.008	-0.086	0.977	0.142	0.998	4.243	0.064	0.335	0.727
HAS	0.033	-0.035	0.009	-0.068	0.991	0.114	0.991	5.183	0.474	0.783	0.863
HSIC	0.067	-0.036	0.008	-0.078	0.983	0.145	0.944	5.345	0.420	0.595	0.785
UNM	0.062	-0.091	0.013	-0.081	0.990	0.170	0.961	5.361	0.272	0.502	0.895
AMG	0.066	-0.055	0.019	-0.131	0.988	0.153	0.973	6.292	0.512	0.698	0.982
RE	0.051	-0.063	0.008	-0.061	0.986	0.178	0.987	4.910	0.065	0.351	0.642
JWN	0.030	-0.009	0.005	-0.080	0.996	0.119	1.013	5.552	0.348	0.777	0.895
PHM	-0.007	0.053	0.008	-0.053	0.997	0.112	1.105	6.760	0.687	0.791	0.865
TMK	0.041	-0.100	0.006	-0.102	0.993	0.159	0.928	8.418	0.517	0.488	0.861

FFIV	0.117	-0.024	0.016	-0.054	0.992	0.085	1.054	4.081	0.229	0.619	0.921
GT	0.058	0.006	0.011	-0.059	0.995	0.111	0.987	5.071	0.172	0.765	0.984
LUK	0.041	-0.037	0.011	-0.045	0.993	0.166	0.986	6.484	0.568	0.551	0.911
PNW	0.014	-0.013	0.008	-0.083	0.980	0.150	0.976	5.303	0.065	0.176	0.943
SEE	0.025	-0.029	0.016	-0.067	0.989	0.136	0.996	3.770	0.539	0.828	0.904
SNA	0.038	-0.010	0.016	-0.110	0.988	0.165	1.024	3.973	0.288	0.586	0.804
UDR	0.049	-0.045	0.006	-0.044	0.995	0.186	0.988	7.603	0.124	0.556	0.664
XRX	0.034	-0.077	0.016	-0.069	0.989	0.113	0.976	4.770	0.446	0.613	0.816
FL	0.093	-0.008	0.017	-0.080	0.989	0.132	1.019	5.102	0.455	0.602	0.738
PWR	0.029	-0.001	0.009	-0.075	0.994	0.120	0.982	5.088	0.136	0.639	0.846
AIZ	0.057	-0.077	0.009	-0.042	0.992	0.155	0.962	4.938	0.479	0.583	0.959
NFX	0.081	0.016	0.013	-0.037	0.993	0.096	0.983	5.850	0.063	0.317	0.816
PDCO	0.065	-0.031	0.006	-0.028	0.992	0.102	0.975	3.929	0.106	0.385	0.584
BRKB	0.028	-0.032	0.009	-0.066	0.984	0.236	1.056	4.875	0.117	0.387	0.783

The table shows the estimation of $AR(1) - EGARCH(1,1)$ - skew Student-t distribution for marginal returns and the goodness of fit test for the standarized residuals. We check the goodness of fit using Kolmogorov-Smirnov test, Anderson-Darling test, Neyman's smooth test of fit. All series passed the test with p-values larger than 0.05.

D Estimation for dynamic one factor copula models

	Gaussian Cop.		Student Cop.		MGSt Cop.		Convergence test	
	Est.	s.e.	Est.	s.e.	Est.	s.e.	Geweke	$\hat{R}$
a_1	0.078	(0.005)	0.067	(0.005)	0.061	(0.006)	0.520	1.00
a_2	0.056	(0.007)	0.043	(0.007)	0.045	(0.008)	-0.597	1.03
a_3	0.106	(0.015)	0.059	(0.011)	0.066	(0.014)	-1.534	1.01
a_4	0.106	(0.013)	0.052	(0.014)	0.051	(0.013)	1.453	1.00
a_5	0.086	(0.006)	0.067	(0.006)	0.069	(0.006)	-0.157	1.01
a_6	0.134	(0.011)	0.034	(0.004)	0.035	(0.004)	-0.464	1.00
a_7	0.096	(0.013)	0.038	(0.006)	0.034	(0.005)	0.176	1.01
a_8	0.146	(0.015)	0.025	(0.004)	0.026	(0.004)	0.128	1.00
a_9	0.066	(0.008)	0.049	(0.010)	0.052	(0.009)	1.121	1.00
a_{10}	0.059	(0.007)	0.045	(0.006)	0.047	(0.007)	0.123	1.02
a_{11}	0.066	(0.008)	0.041	(0.004)	0.047	(0.005)	-1.263	1.00
a_{12}	0.094	(0.009)	0.041	(0.004)	0.042	(0.004)	-0.204	1.01
b_1	0.984	(0.002)	0.985	(0.002)	0.984	(0.003)	-0.454	1.00
b_2	0.982	(0.004)	0.987	(0.005)	0.984	(0.006)	-0.323	1.04
b_3	0.950	(0.013)	0.977	(0.009)	0.970	(0.013)	1.199	1.03
b_4	0.892	(0.021)	0.945	(0.037)	0.956	(0.023)	-1.085	1.00
b_5	0.972	(0.004)	0.976	(0.004)	0.977	(0.004)	-1.714	1.00
b_6	0.857	(0.020)	0.986	(0.003)	0.986	(0.003)	1.647	1.01
b_7	0.926	(0.020)	0.985	(0.005)	0.989	(0.003)	-0.552	1.00
b_8	0.855	(0.028)	0.995	(0.002)	0.995	(0.002)	-1.354	1.00
b_9	0.969	(0.007)	0.975	(0.013)	0.976	(0.009)	-0.562	1.00
b_{10}	0.978	(0.005)	0.985	(0.004)	0.985	(0.005)	0.307	1.03
b_{11}	0.966	(0.008)	0.988	(0.002)	0.985	(0.003)	0.113	1.00
b_{12}	0.952	(0.009)	0.992	(0.002)	0.992	(0.002)	0.630	1.00
ν_1			6.816	(0.215)	23.086	(1.926)	0.222	1.03
ν_2			9.827	(0.606)	13.967	(1.671)	0.452	1.01
ν_3			9.690	(0.644)	17.039	(3.107)	-0.589	1.01
ν_4			8.954	(0.471)	10.022	(0.648)	-0.065	1.01
ν_5			9.292	(0.377)	9.875	(0.436)	1.623	1.01
ν_6			11.870	(0.421)	12.354	(0.460)	1.590	1.00
ν_7			9.457	(0.331)	10.230	(0.399)	2.324	1.00
ν_8			11.001	(0.657)	12.028	(0.819)	0.720	1.03
ν_9			9.706	(0.533)	9.997	(0.629)	1.560	1.03
ν_{10}			10.659	(0.530)	13.994	(1.051)	-0.229	1.01
ν_{11}			8.001	(0.224)	8.750	(0.296)	1.069	1.00

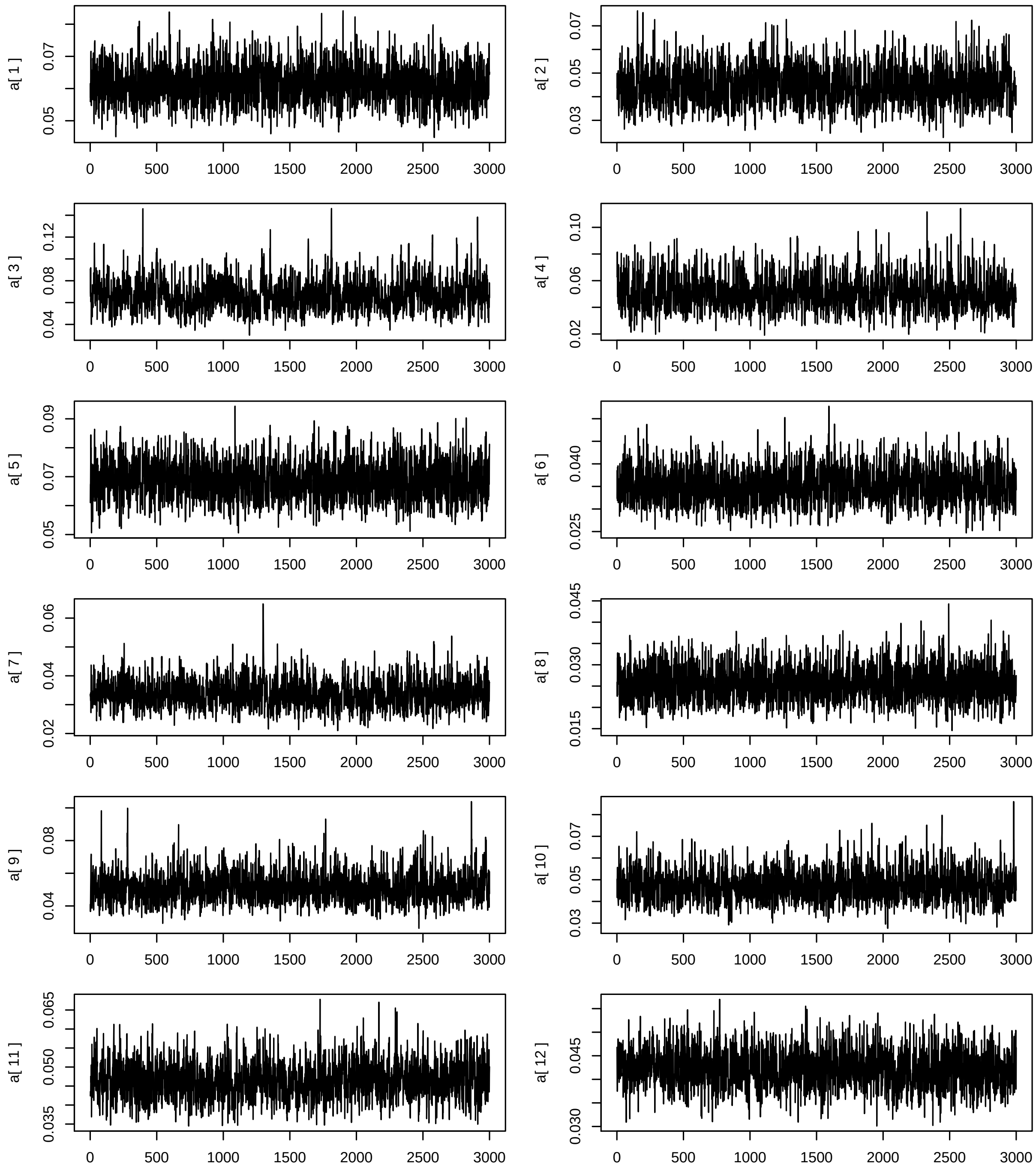
ν_{12}			7.032	(0.186)	7.885	(0.297)	2.025	1.01
γ_1					-1.215	(0.078)	-0.106	1.01
γ_2					-0.450	(0.072)	-0.441	1.02
γ_3					-0.682	(0.124)	0.290	1.00
γ_4					-0.251	(0.032)	-0.523	1.00
γ_5					-0.236	(0.023)	-1.637	1.01
γ_6					-0.264	(0.018)	-1.906	1.00
γ_7					-0.257	(0.020)	-2.264	1.00
γ_8					-0.270	(0.034)	-1.583	1.02
γ_9					-0.246	(0.029)	-2.909	1.03
γ_{10}					-0.418	(0.044)	-0.541	1.01
γ_{11}					-0.212	(0.017)	-1.781	1.01
γ_{12}					-0.184	(0.022)	-1.413	1.00
$f_{1,c}$	1.176	(0.054)	1.226	(0.071)	1.199	(0.075)	0.697	1.00
$f_{2,c}$	2.004	(0.059)	2.233	(0.101)	2.229	(0.104)	0.498	1.00
$f_{3,c}$	1.433	(0.066)	1.496	(0.077)	1.482	(0.072)	1.443	1.00
$f_{4,c}$	1.856	(0.063)	2.069	(0.108)	2.072	(0.105)	0.962	1.00
$f_{5,c}$	1.825	(0.078)	1.960	(0.077)	1.912	(0.079)	0.618	1.00
$f_{6,c}$	2.052	(0.065)	2.211	(0.103)	2.207	(0.107)	0.429	1.01
$f_{7,c}$	1.583	(0.054)	1.693	(0.072)	1.669	(0.083)	-0.568	1.00
$f_{8,c}$	1.854	(0.076)	1.974	(0.078)	1.942	(0.076)	-1.575	1.00
$f_{9,c}$	1.647	(0.049)	1.745	(0.070)	1.719	(0.072)	0.743	1.00
$f_{10,c}$	1.123	(0.063)	1.250	(0.082)	1.236	(0.079)	1.239	1.01
$f_{11,c}$	1.603	(0.057)	1.709	(0.073)	1.690	(0.082)	-0.300	1.00
$f_{12,c}$	1.314	(0.051)	1.401	(0.056)	1.387	(0.058)	0.180	1.00
$f_{13,c}$	1.868	(0.062)	2.068	(0.104)	2.050	(0.102)	0.499	1.01
$f_{14,c}$	1.913	(0.065)	2.031	(0.065)	1.980	(0.070)	0.273	1.00
$f_{15,c}$	1.598	(0.048)	1.697	(0.070)	1.694	(0.072)	-0.178	1.00
$f_{16,c}$	1.281	(0.053)	1.332	(0.077)	1.319	(0.081)	0.168	1.00
$f_{17,c}$	1.976	(0.052)	2.074	(0.074)	2.038	(0.083)	0.614	1.00
$f_{18,c}$	1.206	(0.075)	1.235	(0.085)	1.221	(0.080)	1.275	1.00
$f_{19,c}$	1.145	(0.077)	1.189	(0.077)	1.158	(0.078)	-0.265	1.00
$f_{20,c}$	2.205	(0.055)	2.319	(0.072)	2.287	(0.082)	0.774	1.00
$f_{21,c}$	1.453	(0.052)	1.551	(0.104)	1.515	(0.109)	0.157	1.00
$f_{22,c}$	1.172	(0.066)	1.240	(0.076)	1.219	(0.071)	-0.326	1.00
$f_{23,c}$	2.122	(0.055)	2.257	(0.069)	2.225	(0.082)	0.467	1.00
$f_{24,c}$	1.188	(0.067)	1.239	(0.073)	1.238	(0.069)	-0.414	1.00
$f_{25,c}$	1.869	(0.061)	2.010	(0.103)	2.009	(0.102)	2.733	1.00
$f_{26,c}$	1.930	(0.062)	2.130	(0.105)	2.120	(0.104)	-0.059	1.01
$f_{27,c}$	1.619	(0.052)	1.772	(0.073)	1.747	(0.081)	0.966	1.00
$f_{28,c}$	1.924	(0.049)	2.037	(0.071)	2.004	(0.075)	0.531	1.00
$f_{29,c}$	1.375	(0.056)	1.469	(0.075)	1.460	(0.082)	-0.504	1.01
$f_{30,c}$	1.435	(0.056)	1.498	(0.069)	1.476	(0.080)	0.345	1.00
$f_{31,c}$	1.673	(0.098)	1.747	(0.096)	1.755	(0.092)	-0.162	1.00
$f_{32,c}$	1.985	(0.065)	1.977	(0.103)	1.975	(0.103)	1.208	1.00
$f_{33,c}$	1.525	(0.049)	1.626	(0.071)	1.596	(0.073)	0.174	1.00
$f_{34,c}$	1.947	(0.062)	2.146	(0.102)	2.144	(0.104)	2.017	1.01
$f_{35,c}$	2.045	(0.062)	2.125	(0.105)	2.104	(0.105)	0.323	1.00
$f_{36,c}$	1.856	(0.063)	2.004	(0.103)	2.025	(0.105)	0.084	1.00
$f_{37,c}$	1.902	(0.064)	1.972	(0.102)	1.974	(0.103)	-0.700	1.00
$f_{38,c}$	1.705	(0.065)	1.852	(0.086)	1.852	(0.081)	0.548	1.00
$f_{39,c}$	1.748	(0.077)	1.846	(0.077)	1.813	(0.080)	-0.924	1.00
$f_{40,c}$	1.790	(0.065)	1.908	(0.067)	1.860	(0.068)	0.178	1.00
$f_{41,c}$	1.761	(0.055)	1.887	(0.074)	1.853	(0.082)	0.553	1.00
$f_{42,c}$	1.305	(0.054)	1.359	(0.073)	1.346	(0.082)	-1.482	1.01
$f_{43,c}$	1.638	(0.056)	1.739	(0.075)	1.711	(0.082)	-0.249	1.00
$f_{44,c}$	1.371	(0.064)	1.409	(0.099)	1.388	(0.103)	-0.301	1.00

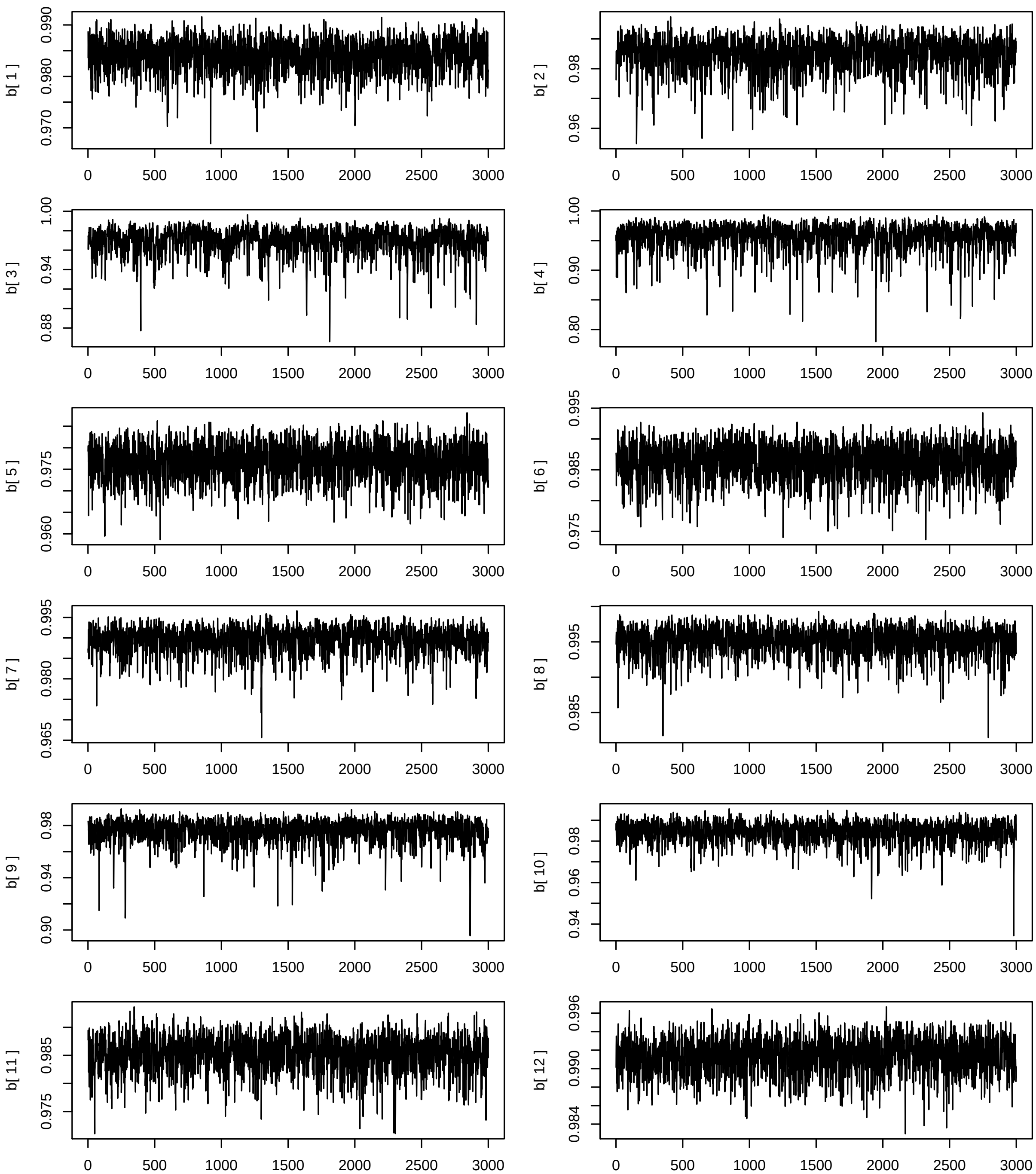
$f_{45,c}$	1.513	(0.050)	1.614	(0.105)	1.578	(0.107)	1.540	1.00
$f_{46,c}$	1.217	(0.052)	1.301	(0.055)	1.290	(0.058)	-0.136	1.00
$f_{47,c}$	1.609	(0.051)	1.750	(0.105)	1.735	(0.109)	0.323	1.00
$f_{48,c}$	1.412	(0.099)	1.576	(0.097)	1.603	(0.090)	-0.013	1.00
$f_{49,c}$	1.106	(0.067)	1.180	(0.076)	1.163	(0.071)	-0.487	1.00
$f_{50,c}$	2.228	(0.063)	2.410	(0.087)	2.408	(0.082)	0.336	1.00
$f_{51,c}$	2.214	(0.064)	2.440	(0.084)	2.432	(0.081)	1.544	1.00
$f_{52,c}$	1.502	(0.057)	1.535	(0.072)	1.517	(0.081)	0.030	1.01
$f_{53,c}$	1.636	(0.094)	1.721	(0.101)	1.724	(0.090)	-0.666	1.00
$f_{54,c}$	1.338	(0.066)	1.487	(0.089)	1.496	(0.079)	1.876	1.00
$f_{55,c}$	1.093	(0.076)	1.200	(0.085)	1.211	(0.083)	-1.365	1.00
$f_{56,c}$	1.692	(0.066)	1.811	(0.088)	1.804	(0.082)	-0.139	1.01
$f_{57,c}$	1.366	(0.076)	1.454	(0.078)	1.432	(0.078)	-1.640	1.00
$f_{58,c}$	1.274	(0.053)	1.324	(0.100)	1.304	(0.107)	0.139	1.00
$f_{59,c}$	1.739	(0.067)	1.823	(0.068)	1.805	(0.071)	-0.898	1.01
$f_{60,c}$	1.738	(0.050)	1.857	(0.054)	1.814	(0.057)	-0.727	1.00
$f_{61,c}$	1.241	(0.075)	1.319	(0.077)	1.291	(0.076)	-0.083	1.00
$f_{62,c}$	1.335	(0.077)	1.397	(0.082)	1.388	(0.081)	-0.625	1.01
$f_{63,c}$	1.875	(0.067)	1.995	(0.070)	1.948	(0.068)	-0.817	1.00
$f_{64,c}$	1.031	(0.066)	1.143	(0.087)	1.148	(0.081)	-0.636	1.00
$f_{65,c}$	2.099	(0.066)	2.222	(0.088)	2.230	(0.082)	0.132	1.00
$f_{66,c}$	1.614	(0.062)	1.658	(0.084)	1.667	(0.080)	0.454	1.00
$f_{67,c}$	1.725	(0.060)	1.769	(0.103)	1.767	(0.106)	1.894	1.00
$f_{68,c}$	1.846	(0.066)	1.997	(0.086)	2.028	(0.082)	2.486	1.00
$f_{69,c}$	1.728	(0.055)	1.880	(0.073)	1.852	(0.081)	0.964	1.00
$f_{70,c}$	1.251	(0.074)	1.316	(0.085)	1.290	(0.081)	-0.927	1.00
$f_{71,c}$	1.268	(0.052)	1.358	(0.073)	1.334	(0.073)	0.387	1.00
$f_{72,c}$	1.097	(0.068)	1.156	(0.077)	1.126	(0.072)	-0.755	1.00
$f_{73,c}$	1.941	(0.053)	2.064	(0.073)	2.030	(0.081)	-0.296	1.00
$f_{74,c}$	1.353	(0.080)	1.377	(0.083)	1.364	(0.081)	0.000	1.00
$f_{75,c}$	0.994	(0.053)	1.035	(0.104)	1.015	(0.107)	1.587	1.00
$f_{76,c}$	1.431	(0.095)	1.542	(0.102)	1.526	(0.090)	-0.163	1.00
$f_{77,c}$	2.369	(0.064)	2.442	(0.108)	2.422	(0.103)	-0.427	1.00
$f_{78,c}$	1.440	(0.077)	1.544	(0.074)	1.520	(0.081)	-0.097	1.00
$f_{79,c}$	1.502	(0.098)	1.596	(0.099)	1.635	(0.091)	-1.353	1.00
$f_{80,c}$	1.286	(0.068)	1.324	(0.076)	1.287	(0.073)	1.083	1.01
$f_{81,c}$	1.635	(0.048)	1.760	(0.069)	1.731	(0.074)	-0.653	1.00
$f_{82,c}$	1.129	(0.079)	1.220	(0.088)	1.233	(0.083)	2.517	1.00
$f_{83,c}$	1.196	(0.077)	1.281	(0.078)	1.285	(0.082)	0.388	1.00
$f_{84,c}$	1.661	(0.077)	1.758	(0.074)	1.702	(0.077)	-0.718	1.00
$f_{85,c}$	1.761	(0.050)	1.853	(0.070)	1.822	(0.074)	-1.046	1.00
$f_{86,c}$	1.565	(0.074)	1.587	(0.084)	1.555	(0.084)	0.712	1.00
$f_{87,c}$	1.127	(0.078)	1.170	(0.085)	1.156	(0.080)	-0.532	1.00
$f_{88,c}$	1.297	(0.066)	1.322	(0.100)	1.309	(0.102)	0.157	1.00
$f_{89,c}$	1.359	(0.079)	1.455	(0.082)	1.455	(0.082)	1.047	1.01
$f_{90,c}$	1.836	(0.068)	1.935	(0.068)	1.894	(0.070)	1.154	1.00
$f_{91,c}$	0.965	(0.066)	1.030	(0.084)	1.021	(0.080)	0.422	1.00
$f_{92,c}$	1.642	(0.048)	1.690	(0.105)	1.628	(0.115)	0.631	1.01
$f_{93,c}$	1.447	(0.094)	1.547	(0.093)	1.400	(0.090)	-1.945	1.00
$f_{94,c}$	1.600	(0.078)	1.670	(0.075)	1.617	(0.079)	-0.423	1.00
$f_{95,c}$	2.092	(0.050)	2.183	(0.071)	2.162	(0.072)	-0.350	1.00
$f_{96,c}$	1.287	(0.066)	1.367	(0.069)	1.338	(0.071)	0.184	1.00
$f_{97,c}$	1.288	(0.068)	1.335	(0.075)	1.287	(0.071)	-1.250	1.00
$f_{98,c}$	2.329	(0.065)	2.381	(0.089)	2.368	(0.083)	0.253	1.00
$f_{99,c}$	1.214	(0.080)	1.280	(0.086)	1.268	(0.082)	1.802	1.00
$f_{100,c}$	1.149	(0.078)	1.220	(0.077)	1.186	(0.078)	0.996	1.00
$f_{101,c}$	1.737	(0.052)	1.803	(0.071)	1.772	(0.073)	-2.006	1.01

$f_{102,c}$	1.432	(0.102)	1.552	(0.100)	1.385	(0.090)	0.919	1.00
$f_{103,c}$	1.970	(0.051)	2.055	(0.057)	2.000	(0.058)	-1.244	1.00
$f_{104,c}$	1.596	(0.080)	1.691	(0.074)	1.655	(0.079)	-1.200	1.00
$f_{105,c}$	1.521	(0.051)	1.619	(0.072)	1.575	(0.074)	-1.049	1.00
$f_{106,c}$	1.464	(0.051)	1.566	(0.071)	1.537	(0.072)	0.194	1.00
$f_{107,c}$	1.829	(0.066)	1.904	(0.084)	1.893	(0.082)	0.308	1.00
$f_{108,c}$	1.391	(0.097)	1.550	(0.101)	1.469	(0.089)	0.242	1.00
$f_{109,c}$	1.553	(0.067)	1.630	(0.066)	1.614	(0.070)	-0.375	1.01
$f_{110,c}$	1.609	(0.100)	1.709	(0.100)	1.744	(0.089)	0.369	1.00
$f_{111,c}$	1.693	(0.065)	1.731	(0.102)	1.730	(0.101)	0.576	1.00
$f_{112,c}$	1.728	(0.052)	1.765	(0.114)	1.749	(0.118)	1.558	1.00
$f_{113,c}$	1.965	(0.049)	2.120	(0.056)	2.089	(0.057)	0.425	1.00
$f_{114,c}$	1.545	(0.098)	1.551	(0.098)	1.452	(0.089)	-1.053	1.00
$f_{115,c}$	1.331	(0.077)	1.414	(0.083)	1.412	(0.081)	1.208	1.00
$f_{116,c}$	1.706	(0.048)	1.753	(0.068)	1.715	(0.074)	-0.851	1.00
$f_{117,c}$	1.361	(0.076)	1.458	(0.075)	1.416	(0.077)	0.826	1.01
$f_{118,c}$	1.112	(0.051)	1.197	(0.057)	1.181	(0.058)	0.135	1.01
$f_{119,c}$	1.362	(0.054)	1.390	(0.104)	1.339	(0.109)	-1.236	1.00
$f_{120,c}$	1.644	(0.073)	1.716	(0.083)	1.712	(0.079)	-0.922	1.00
$f_{121,c}$	2.074	(0.063)	2.242	(0.084)	2.250	(0.081)	1.134	1.00
$f_{122,c}$	2.229	(0.061)	2.267	(0.109)	2.250	(0.107)	1.495	1.00
$f_{123,c}$	1.356	(0.064)	1.410	(0.087)	1.435	(0.082)	1.006	1.00
$f_{124,c}$	1.558	(0.073)	1.584	(0.081)	1.556	(0.081)	-0.127	1.00
$f_{125,c}$	1.491	(0.101)	1.607	(0.095)	1.476	(0.091)	0.073	1.00
$f_{126,c}$	2.189	(0.064)	2.383	(0.091)	2.392	(0.080)	0.000	1.00
$f_{127,c}$	1.261	(0.051)	1.348	(0.071)	1.331	(0.072)	-0.852	1.01
$f_{128,c}$	1.555	(0.050)	1.631	(0.070)	1.612	(0.073)	-0.682	1.00
$f_{129,c}$	2.099	(0.083)	2.074	(0.087)	2.010	(0.085)	-0.916	1.00
$f_{130,c}$	1.376	(0.075)	1.456	(0.079)	1.458	(0.081)	0.482	1.01
$f_{131,c}$	1.806	(0.050)	1.883	(0.055)	1.848	(0.057)	-0.529	1.00
$f_{132,c}$	2.065	(0.048)	2.217	(0.070)	2.176	(0.073)	-1.100	1.00
$f_{133,c}$	1.560	(0.064)	1.612	(0.103)	1.599	(0.102)	0.056	1.00
$f_{134,c}$	1.705	(0.050)	1.837	(0.070)	1.818	(0.071)	0.152	1.00
$f_{135,c}$	1.233	(0.074)	1.264	(0.082)	1.242	(0.082)	-1.236	1.00
$f_{136,c}$	1.517	(0.099)	1.549	(0.098)	1.445	(0.089)	1.552	1.01
$f_{137,c}$	1.677	(0.063)	1.805	(0.084)	1.811	(0.080)	1.030	1.00
$f_{138,c}$	1.386	(0.098)	1.458	(0.098)	1.494	(0.092)	-0.797	1.00
$f_{139,c}$	1.598	(0.073)	1.662	(0.081)	1.663	(0.083)	-0.671	1.01
$f_{140,c}$	1.754	(0.063)	1.755	(0.086)	1.752	(0.081)	-0.664	1.01

The table shows the estimation of one factor copula models and the convergence check for MGSt copulas using the Geweke statistics (Geweke (1992)) and the Gelman and Rubin's convergence statistics ($\hat{R}$ statistics, Gelman and Rubin (1992)). We report the Geweke statistic based on the standard Z-score of the difference between the two sample means of first 10% proportion and last 50% proportion of posterior samples. We report the 0.95% upper confidence limits statistics, see Plummer et al. (2006). The $\hat{R}$ values are close to 1 indicate the convergence.

E The trace plots of parameters in MGSt copulas

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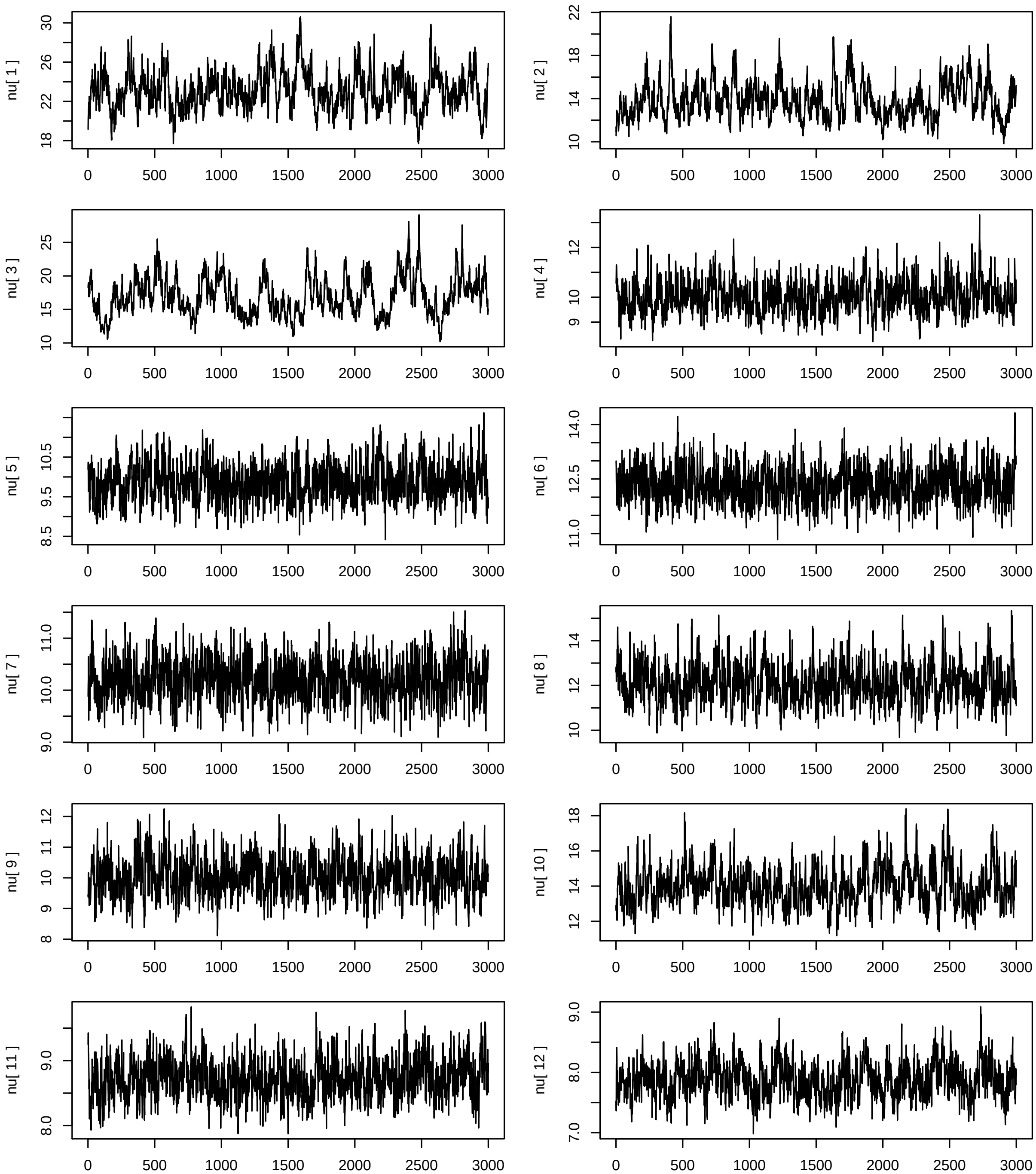
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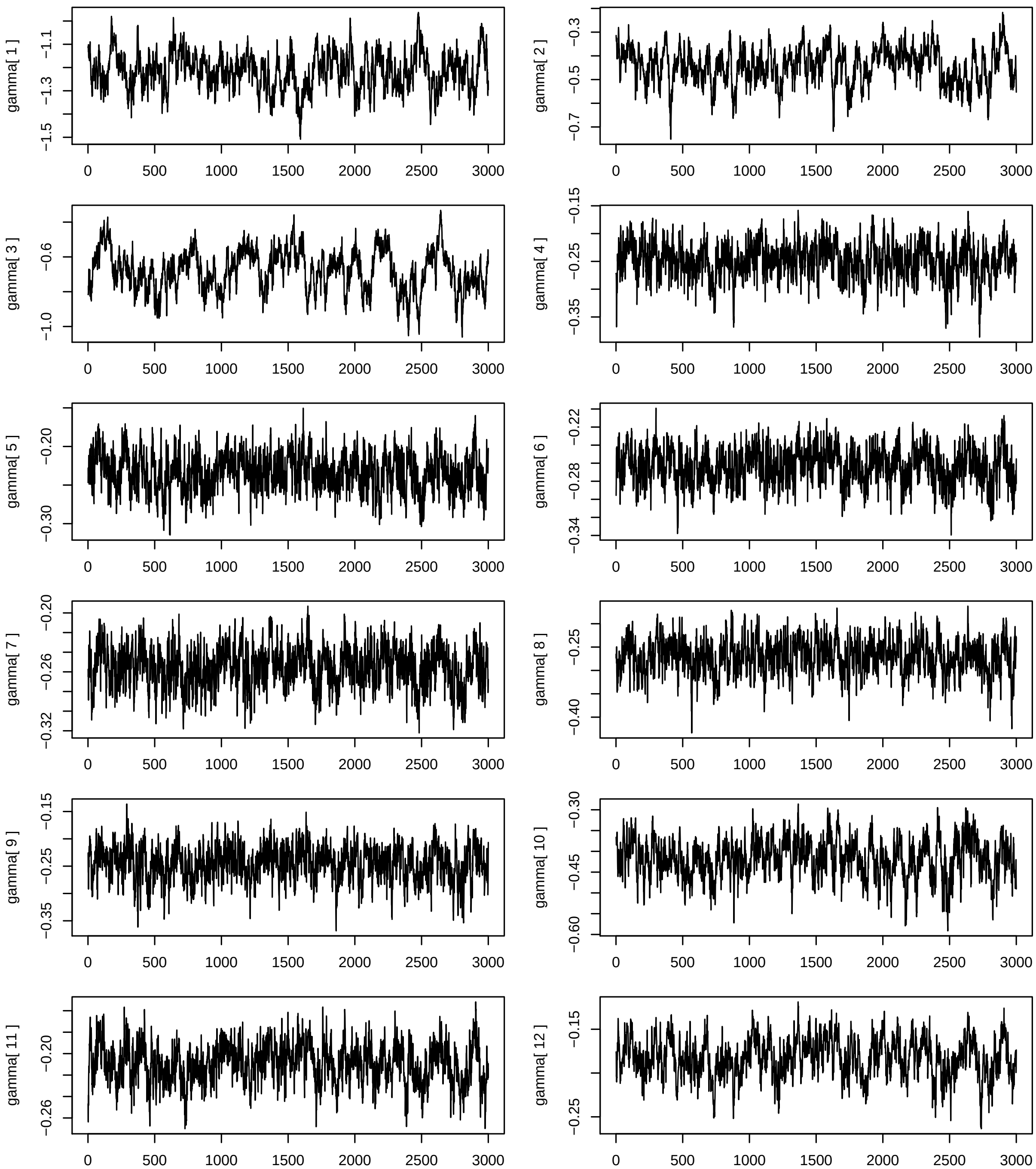
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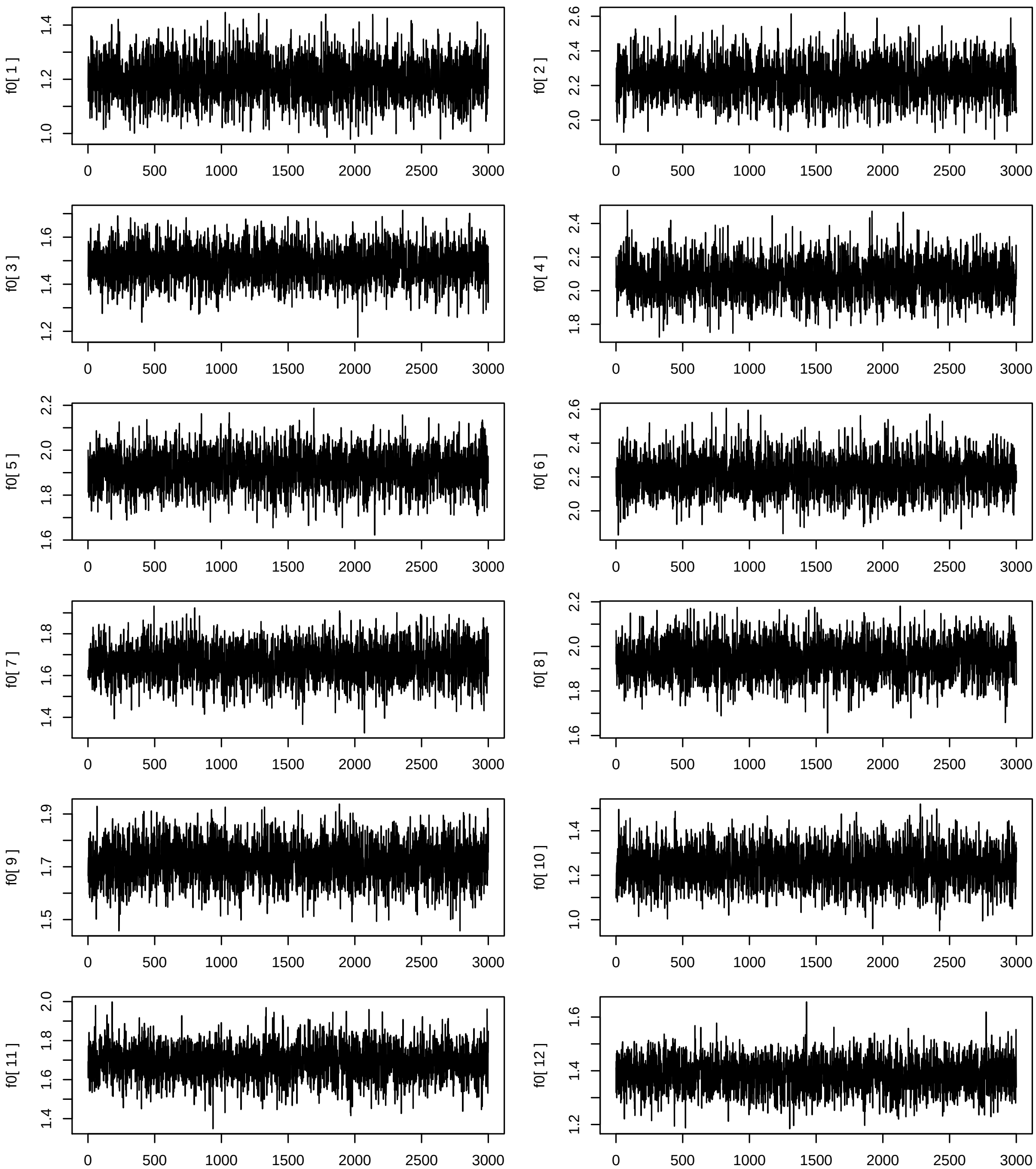
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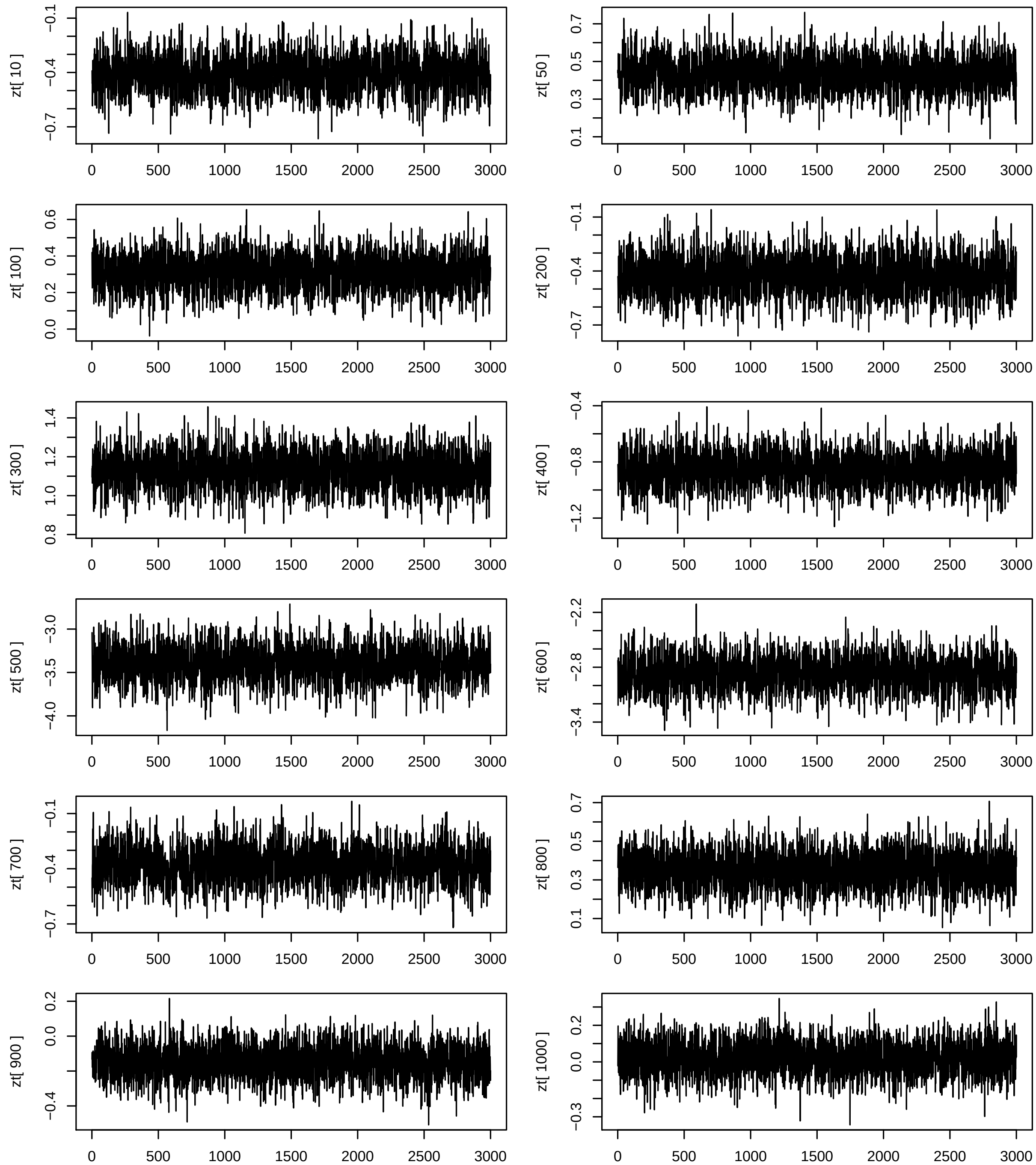

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